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Alpha Particle Dynamics in Muon-Boosted Fusion Propulsion System

Terry Kammash[1], Ricky Tang[2] and Alec D. Gallimore[3]
University of Michigan, Ann Arbor, MI, 48109

In a previous paper,[1] we demonstrated that negative muons resulting from antiproton annihilation in a relatively cold deuterium-tritium (DT) plasma confined in a gasdynamic mirror (GDM) can result in catalyzing on average over 100 fusion reactions. The alpha particles produced by these reactions could contribute significantly to heating the background plasma toward ignition. In fact, it was pointed out that on the basis of energetics only, muon-catalyzed fusion would reduce the amount of antiprotons required to achieve thermonuclear burn by about 60%. This scenario, however, does not address the issue of alpha particle confinement in the GDM, and thereby leaves open the question of their true effectiveness in providing the heating noted above. In this paper, we address this problem by noting that, as they slow down, these alpha particles can escape from the system. We deduce explicit expressions for alpha particle density as a function of energy, and calculate the mean energy of these particles allowing simultaneously for slowing down and escape as reflected by the confinement time. Assuming that the alpha particles slow down primarily on the electrons, as is the case in relatively cold plasmas, we find that muon catalyzed fusion is indeed effective in heating the plasma in a GDM device.

Nomenclature

c_μ	=	number of catalyzed fusion per negative muon
E	=	alpha particle energy
E_0	=	initial (birth) energy of alpha particle
E_{th}	=	thermal energy
$\overline{E}_\alpha$	=	mean alpha particle energy
e	=	elementary charge
L	=	plasma length
$\ln\Lambda$	=	Coulomb logarithm
m	=	particle mass
N_a	=	confined alpha particle density
n	=	number density
$n_\alpha(E)$	=	alpha particle energy distribution function
R	=	plasma mirror ratio
T_e	=	electron temperature
τ	=	time constant, or confinement time
τ_μ	=	muon lifetime
v	=	monoenergetic particle velocity
Z	=	particle charge state

[1] Stephen S. Attwood Professor Emeritus of Nuclear Engineering and Radiological Sciences, 2355 Bonisteel Blvd., Ann Arbor, MI 48109, AIAA Associate Fellow.
[2] Graduate Student, Dept. of Aerospace Engineering, 1320 Beal Ave., Ann Arbor, MI 48109, AIAA Member.
[3] Arthur F. Thurnau Professor of Aerospace Engineering, 1320 Beal Ave., Ann Arbor, MI 48109, AIAA Associate Fellow.

I. Introductory Remarks

We have shown in previous work[1] that in antiproton-driven fusion propulsion systems, plasma heating results from fission fragments as well as from the annihilation products produced by antiproton annihilation in U^{238} targets. We have also indicated that some additional significant heating comes about as a result of muon catalysis in a deuterium-tritium (DT) plasma whereby a negative muon can uniquely attach itself to both D and T ions, thereby allowing them to undergo fusion reactions and releasing energetic alpha particles into the plasma. Such catalysis results in more than 100 fusion reactions during the lifetime of the muon, and the resulting alpha particles provide sizable amount of heating towards ignition. The assumption made in those studies is that the alpha particles in the confinement device – the gasdynamic mirror (GDM) – deposit their energy through collisions and not escape while doing so. In this paper, we address this question by allowing for escape while these particles slow down. We assume that alpha particle confinement follows that of the ions, and deduce the appropriate expressions for their velocity distribution and mean energy, as well as their confinement time in the GDM. We find that despite particle losses, the fraction of alpha particles that are confined can still contribute a significant amount of plasma heating as suggested earlier. What follows is a mathematical confirmation of these predictions.

II. Energy Distribution

The number of alpha particles in an interval of energy ΔE is $n_\alpha(E)\Delta E$, where $n_\alpha(E)$ is the number density per unit energy. If a loss mechanism has a time constant $\tau_s(E)$, then in steady state we have

$$\frac{d}{dt}\left[n_\alpha(E)\Delta E\right] = n_\alpha(E)\left(\frac{dE}{dt}\right)_E - n_\alpha(E+\Delta E)\left(\frac{dE}{dt}\right)_{E+\Delta E} - \frac{n_\alpha(E)\Delta E}{\tau(E)} = 0 \tag{1}$$

Rearranging and using the definition of a derivative, we obtain the following governing differential equation for the energy distribution of alpha particles.

$$\frac{\partial}{\partial E}\left(n_\alpha \frac{dE}{dt}\right) + \frac{n_\alpha(E)}{\tau(E)} = \frac{\partial n_\alpha}{\partial E}\left(\frac{dE}{dt}\right) + n_\alpha(E)\frac{\partial}{\partial E}\left(\frac{dE}{dt}\right) + \frac{n_\alpha(E)}{\tau(E)} = 0 \tag{2}$$

Integrating Eq. (2) over the range of energies E to E_0, where E_0 is the initial energy (i.e. birth energy) of the alpha particles, we obtain the following.

$$n_\alpha(E) = n_\alpha(E_0)\exp\left\{\int_E^{E_0}\left[\frac{\frac{\partial}{\partial E}\left(\frac{dE}{dt}\right)}{dE/dt} + \frac{1}{\tau(E)\frac{dE}{dt}}\right]dE\right\} \tag{3}$$

The first term of the integrand can be rewritten as follows

$$\frac{\frac{\partial}{\partial E}\left(\frac{dE}{dt}\right)}{dE/dt} = \frac{d}{dE}\left[\ln\left(\frac{dE}{dt}\right)\right] \tag{4}$$

and can be readily integrated yielding the following expression for the alpha particle energy distribution.

$$n_\alpha(E) = n_\alpha(E_0)\frac{\left(dE/dt\right)_{E_0}}{dE/dt}\exp\left\{\int_E^{E_0}\frac{1}{\tau(E)\,dE/dt}dE\right\} \tag{5}$$

Now consider alpha particles produced via muon-catalyzed fusion, the initial energy distribution would be given by the following.

$$n_\alpha(E_0) = \frac{(c_\mu n_\mu)/\tau_\mu}{-\left(dE/dt\right)_{E_0}} \tag{6}$$

where n_μ is the negative muon number density, τ_μ the muon lifetime, and c_μ is the number of catalyzed fusion (i.e. number of alpha particles born) per negative muon. dE/dt represents the rate of decrease of alpha particle energy and can be expressed by the following, with the first term denoting energy loss to the plasma electrons and the second term loss to the ions due to Coulomb collisions.

$$\frac{dE}{dt} = -\left(c_1 E + \frac{c_2}{\sqrt{E}}\right) \tag{7}$$

c_1 and c_2 are coefficients that depend on the incident and target particles, as well as plasma density and temperature.

$$c_1 = \frac{8}{3}\sqrt{2\pi} Z^2 Z_e^2 e^4 (\ln\Lambda) \frac{\sqrt{m_e}}{T_e^{3/2}} \frac{n_e}{m} C_a \qquad \left[\mathrm{s}^{-1}\right] \tag{8a}$$

$$C_a = \left(1.6022\times 10^{-9}\right)^{-3/2} \left(\frac{\mathrm{keV}}{\mathrm{erg}}\right)^{3/2} \tag{8b}$$

where *m* and m_e are respectively the mass of the incident particle (i.e. alpha particle) and the electron. Similarly, *Z* and Z_e are the charge state of the incident particle (i.e. alpha particle) and the electron, respectively. n_e is the electron density, and T_e is the electron temperature. Equation (8a) is written in the CGS system, and all the quantities have the standard CGS units, with the exception of the electron temperature T_e. For convenience, T_e in Eq. (8a) has unit **keV**. The conversion factor C_a makes explicit the conversion to the CGS system. Finally, ln Λ is the Coulomb Logarithm given by the following for a DT plasma,

$$\ln\Lambda = 24 - \log\frac{\sqrt{n_e\left[\mathrm{cm}^{-3}\right]}}{T_e\left[\mathrm{eV}\right]} \tag{9}$$

Similarly for c_2, Eq. (10a) is written in the CGS system, and all quantities have their standard CGS units. The conversion factor C_b ensures that c_2 has the correct energy unit of keV in order to be consistent with the other equations.

$$c_2 = \frac{4\pi Z^2 Z_i^2 e^4 (\ln\Lambda) n_i}{m_i} \sqrt{\frac{m}{2}} C_b \qquad \left[\frac{\mathrm{keV}^{3/2}}{\mathrm{s}}\right] \tag{10a}$$

$$C_b = \left(1.6022\times 10^{-9}\right)^{-3/2} \left(\frac{\mathrm{keV}}{\mathrm{erg}}\right)^{3/2} \tag{10b}$$

Substituting Eqs. (6) and (7) into Eq. (5) yields the following energy distribution for alpha particles produced via muon-catalyzed fusion, where *E* has unit **keV**.

$$n_\alpha(E) = \frac{(c_\mu n_\mu)/\tau_\mu}{c_1 E + \frac{c_2}{\sqrt{E}}} \exp\left\{-\int_E^{E_0} \frac{\sqrt{E}}{\left(c_1 E^{3/2} + c_2\right)\tau(E)} dE\right\} \tag{11}$$

To evaluate Eq. (11), we need the time constant (i.e. confinement time) $\tau_s(E)$. The confinement time for the GDM, ignoring the ambipolar potential, is as follows,

$$\tau(E) = \frac{RL}{v} \tag{12}$$

Here *R* is the plasma mirror ratio, which is the ratio of the magnetic field seen by the plasma at the mirror to that at the center. The monoenergetic particle velocity is given by Eq. (13)

$$v = \sqrt{\frac{2EC_c}{m_\alpha}} \tag{13a}$$

$$C_c = 1.6022 \times 10^{-9} \text{ erg/keV} \tag{13b}$$

where m_α is the mass of the alpha particle, and C_c is a unit conversion factor allowing *E* in Eq. (13a) to be expressed in **keV** in order to be consistent with Eq. (11). All the other quantities in Eqs. (12) and (13a) have the standard CGS units. The choice of the CGS system here is arbitrary; SI units may be used instead in Eqs. (12) and (13a), in which case C_c would be modified to relate Joule to keV. Substituting these equations into Eq. (11) yields the final expression for the energy distribution for alpha particles produced via muon-catalyzed fusion inside the GDM.

$$n_\alpha(E) = \frac{(c_\mu n_\mu)/\tau_\mu}{c_1 E + \frac{c_2}{\sqrt{E}}} \exp\left\{ -\sqrt{\frac{2C_c}{m_\alpha}} \frac{1}{RL} \int_E^{E_0} \frac{E}{c_1 E^{3/2} + c_2} dE \right\} \quad \left[\text{cm}^{-3} \cdot \text{keV}^{-1}\right] \tag{14}$$

III. Electron Heating Only

The analytical solution to the full integral in Eq. (14) is very complicated. For a relatively cold plasma, e.g. at the ionization temperature, c_1 can be several orders of magnitude larger than c_2, and therefore, electron heating dominates, which is what we expect for a cold plasma. If we envision a GDM system wherein these alpha particles produced via muon-catalyzed DT fusion reactions contribute to the initial phase of the plasma heating, it is reasonable to assume the bulk of their energy being deposited into the plasma electrons. We can therefore simplify the integral in Eq. (14) accordingly by assuming $c_2 = 0$.

$$n_\alpha(E) = \frac{(c_\mu n_\mu)/\tau_\mu}{c_1 E} \exp\left\{ -\sqrt{\frac{2C_c}{m_\alpha}} \frac{1}{RL} \int_E^{E_0} \frac{dE}{c_1 E^{1/2}} \right\} \tag{15}$$

The resulting integral can be easily evaluated, yielding the following distribution function.

$$\begin{aligned} n_\alpha(E) &= \frac{c_\mu n_\mu}{c_1 E \tau_\mu} \exp\left\{ -\sqrt{\frac{2C_c}{m_\alpha}} \frac{2}{RLc_1} \left(\sqrt{E_0} - \sqrt{E}\right) \right\} \\ &= \frac{c_\mu n_\mu}{c_1 E \tau_\mu} \exp\left\{ -A\left(\sqrt{E_0} - \sqrt{E}\right) \right\} \end{aligned} \tag{16}$$

where we have defined the following quantity.

$$A \equiv \sqrt{\frac{2C_c}{m_\alpha}} \frac{2}{RLc_1} > 0 \quad \left[\text{keV}^{-1/2}\right] \tag{17}$$

Inspecting Eq. (16), we see that the distribution behaves as follows.

$$n_\alpha(E) \sim \frac{e^{A\sqrt{E}}}{E} \tag{18}$$

For relatively small energy E, $1/E$ dominates, whereas for large E, the exponential dominates. The minimum of the distribution occurs at an energy $E_{\min}$.

$$E_{\min} = \frac{4}{A^2} \tag{19}$$

Since the energy of the alpha particles will be bounded by E_{th} (thermal energy) and E_0 (initial energy at birth, i.e. 3.5 MeV), only this portion of $n_\alpha(E)$ is meaningful. Using typical orders of magnitude for the defined quantity A, i.e. 10^{-3} to 10^{-5} for a dense cold plasma, we see that our distribution lies significantly to the left of the minimum, where $1/E$ dominates. Figure 1 shows a representative plot of the distribution function for this range of energies.

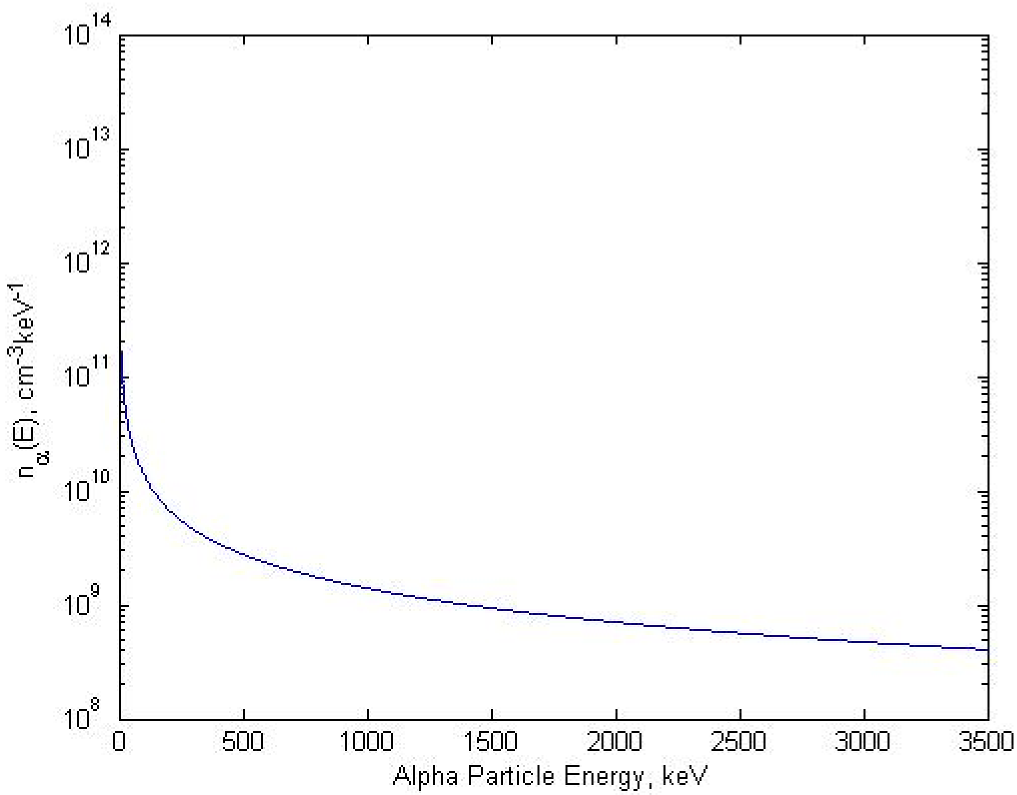

Figure 1. Typical profile for the alpha particle energy distribution function.

A. Confined Alpha Particle Density

To obtain the total density, we integrate the distribution, Eq. (16), over all energies between the lower and upper bounds.

$$N_\alpha = \int_{E_{th}}^{E_0} n_\alpha(E)dE = \frac{c_\mu n_\mu}{c_1 \tau_\mu} e^{-A\sqrt{E_0}} \int_{E_{th}}^{E_0} \frac{e^{A\sqrt{E}}}{E} dE \tag{20}$$

A change of variable transforms the integral in Eq. (20) into

$$N_\alpha = \frac{2c_\mu n_\mu}{c_1\tau_\mu} e^{-A\sqrt{E_0}} \int_{-A\sqrt{E_{th}}}^{-A\sqrt{E_0}} \frac{e^{-z}}{z} dz$$
$$= \frac{2c_\mu n_\mu}{c_1\tau_\mu} e^{-A\sqrt{E_0}} \left[- \int_{-A\sqrt{E_0}}^{\infty} \frac{e^{-z}}{z} dz + \int_{-A\sqrt{E_{th}}}^{\infty} \frac{e^{-z}}{z} dz \right] \tag{21}$$

Each of the integrals in Eq. (21) is defined as the "exponential integral function" and is denoted by Ei. Therefore, the total density assuming electron heating only is given by the following.

$$N_\alpha = \frac{2c_\mu n_\mu}{c_1\tau_\mu} e^{-A\sqrt{E_0}} \left[Ei\left(A\sqrt{E_0}\right) - Ei\left(A\sqrt{E_{th}}\right) \right] \quad \left[\text{cm}^{-3}\right] \tag{22}$$

B. Mean Kinetic Energy

The mean alpha energy can be calculated as follows.

$$\overline{E}_\alpha = \frac{\int_{E_{th}}^{E_0} E n_\alpha(E) dE}{N_\alpha} = \frac{\frac{c_\mu n_\mu}{c_1\tau_\mu} e^{-A\sqrt{E_0}} \int_{E_{th}}^{E_0} e^{A\sqrt{E}} dE}{N_\alpha} \equiv \frac{I}{N_\alpha} \tag{23}$$

The integral I can be readily evaluated by first making a variable substitution $x \equiv A\sqrt{E}$ and then using integration by parts.

$$I = \frac{2c_\mu n_\mu}{A^2 c_1\tau_\mu} e^{-A\sqrt{E_0}} \left[e^{A\sqrt{E_0}} \left(A\sqrt{E_0} - 1\right) - e^{A\sqrt{E_{th}}} \left(A\sqrt{E_{th}} - 1\right) \right] \tag{24}$$

And the final expression for the mean alpha particle energy is

$$\overline{E}_\alpha = \frac{e^{A\sqrt{E_0}} \left(A\sqrt{E_0} - 1\right) - e^{A\sqrt{E_{th}}} \left(A\sqrt{E_{th}} - 1\right)}{A^2 \left[Ei\left(A\sqrt{E_0}\right) - Ei\left(A\sqrt{E_{th}}\right) \right]} \quad [\text{keV}] \tag{25}$$

C. Sample Calculations

We consider a deuterium-tritium (DT) plasma with density 5×10^{16} cm^{-3} at an initial temperature of 13.6 eV, corresponding to the ionization potential of the propellant. The system is antiproton driven. First, "at rest" annihilation of antiprotons in uranium-238 targets causes fission at nearly 100% efficiency.[2,3] The resulting fission fragments and annihilation products, namely pions and their decay product muons, contribute to the heating of the plasma. In addition, in a DT plasma (even if it is cold), each negative muon can on average catalyze approximately 100 DT fusion reactions, each releasing an alpha particle of 3.5 MeV of kinetic energy that further contributes to the initial phase of plasma heating.

For a GDM with plasma mirror ratio of 25, a plasma length of 2 meters (note: this calculation is not sensitive to these two quantities), and an antiproton density of 2.07×10^{12} cm^{-3} (based upon heating requirements not addressed in this paper and corresponding conservatively to the same density of negative muons, which in turns yields an initial alpha particle density of 2.07×10^{14} cm^{-3}), the number of alpha particles being confined is about 1.68×10^{13} cm^{-3}. The mean energy of these confined alpha particles is roughly 294 keV.

To determine the change in plasma temperature, we consider a simple energy balance.

$$\frac{3}{2} n_e \left(T_e - T_{e0}\right) = n_{inc} \left(E_0 - E\right) \tag{26}$$

where n_{inc} is the incident particle density (i.e. alpha particles in the current analysis), and the subscript 0 denotes initial values.

If we assume the confined alpha particles deposit almost all of its energy into the plasma electrons, i.e. slow down on the electrons from their birth energy of 3.5 MeV to a final average kinetic energy $E = (3/2)T = 20.4\,\text{eV}$, corresponding to a temperature of 13.6 eV, the change in the electron temperature is $\Delta T_e = 784\,\text{eV}$. This represents the maximum heating produced by these alpha particles.

Alternately, if we make a more conservative estimate and assume the confined alpha particles slow down on the electrons until they reach their mean kinetic energy of 294 keV, then the corresponding change in the electron temperature is $\Delta T_e = 718\,\text{eV}$.

From Eq. (26), we see that

$$\Delta T_e \sim \frac{n_{inc}}{n_e} \Delta E \tag{27}$$

Since ΔE is more or less fixed, the important factor is the density ratio. Increasing this ratio either by increasing antiproton density or decreasing electron density or both can result in a ΔT_e of multi-keV's, for instance. Of course, due to the heating requirements, these two densities are not necessarily independent. For example, changing the electron density will change the minimum antiproton density required, meaning the negative muon density, and hence the confined alpha particle density, will change as well. However, one can increase the antiproton density beyond the minimum required value dictated by the heating requirements to produce a larger ΔT_e, if this is desired and the associated increase in cost of obtaining and confining the antiprotons is not prohibitive.

IV. Conclusion

In this paper, we have derived an energy distribution (or equivalently a velocity distribution) for the alpha particles produced via muon-catalyzed fusion inside the GDM. The distribution incorporates a time constant to address particle losses due to escape from the system during slowing down. We have derived from the distribution the number density for the confined alpha particles, as well as their mean energy, by assuming that the majority of energy transfer is to the plasma electrons, a valid assumption for a relatively cold plasma.

We found that although there are particle losses, the number of alpha particles remaining and the heating they contribute are nevertheless significant. For a given plasma density, we can increase the antiproton density to increase the amount of heating as a result of an increase in the confined alpha particle density. For instance, in the above calculations, doubling the amount of antiprotons will result in ~1.5 keV increase in the electron temperature. The mean energy and the percentage of alpha particles confined, however, will remain the same for a given plasma density.

Another way to increase the contributed heating is to increase the alpha particles utilization. Brief calculations showed that utilization increases as the plasma density decreases. For instance, when $n_e = 1\times10^{16}\ \text{cm}^{-3}$, the percentage of alpha particles confined long enough to heat the plasma increases 4-fold compared to the calculations in the previous section, and the associated heating increases significantly as well. The tradeoff, however, is that the plasma dynamics inside the GDM dictates a rather rapid increase in the plasma length with decreasing plasma density, and the system soon becomes prohibitively massive.

Acknowledgments

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References

[1]Kammash, T., and Tang, R., "Muon-Boosted Fusion Propulsion System," *43rd Joint Propulsion Conference*, Cincinnati, OH, 2007. AIAA-2007-5609.

[2]Hofmann, P., et al., "Fission of Heavy Nuclei Induced by Stopped Antiprotons. I. Inclusive Characteristics of Fission Fragments," *Physical Review C*, Vol. 49, 1994, pp. 2555-2568.

[3]Kim, Y. S., et al., "Fission of Heavy Nuclei Induced by Stopped Antiprotons. II. Correlations between Fission Fragments," *Physical Review C*, Vol. 54, 1996, pp. 2469-2476.