

# Density and Spectral Measurements using a 34 GHz Interferometry System

Christopher N. Davis<sup>\*</sup>, Brian E. Gilchrist<sup>§</sup>, and Alec D. Gallimore<sup>¥</sup>

## Abstract

Interferometry was used to investigate the impact of the plasma plume of the P5 thruster on microwave signals. Measurements were made at 17 GHz similar to other measurements made on Hall thrusters [Fr,Sn] and also at 34 GHz using a newly built Ka-band interferometer. Very close to the thruster's exit plane (.05 m) a peak phase shift of 32 degrees at 34 GHz was measured and a peak phase shift of 60 degrees at 17 GHz was measured. At 0.66m from the P5 exit plane, the peak phase shift measured was 16 degrees at 34 GHz and 9 degrees at 17 GHz. Signal attenuation measurements showed at .05 m from the thruster's exit plane around 1.2 dB of signal attenuation for 17 GHz and about 0.5 dB of attenuation for 34 GHz on centerline of the thruster. Electron densities profiles were found using Abel inversion on the phase shift measurements. At a discharge voltage of 300 V and a discharge current of 10 A, a peak electron density of about  $4e10 \text{ cm}^{-3}$  was found .05 m from the exit plane. At 300 V, 5 A, and .05 m from the exit plane, a peak density of  $2.2e10 \text{ cm}^{-3}$  was found. Comparing 17 GHz and 34 GHz shows that a better resolution is achieved using the 34 GHz system in terms of measuring the width of the electron density and phase shift profiles. Spectral measurements of a signal propagating through the plasma plume were also made showing that the presence of harmonics is strongly dependent on operating conditions.

## Nomenclature

$n_c = (f/8.98)^2$ , Critical or cutoff density  
 $r$  = Radial distance from center of the plume  
 $R$  = Maximum plume radius  
 $\lambda$  = Wavelength of a wave  
 $n$  = Electron density  
 $\theta$  = Phase of a wave

using a Ku-band interferometer. The advantages that a Ka-band system has over a Ku-band is that it will allow the measurement of denser plasmas and give better measurement resolution as will be shown in this paper. This study will also map the electron density profiles of the P5 thruster currently in use at the University of Michigan using both a Ka and Ku-band interferometer.

## Introduction

Ku –band Microwave interferometers have been used in the past as accurate, non-intrusive diagnostic tools in the plasma plumes of Hall thrusters [1,2]. A microwave signal passing through a plasma plume can experience both phase shifts and attenuation. Microwave interferometry also provides a non-intrusive method of measuring the electron density of a plasma. This paper will present measurements taken using a new Ka-band interferometer and will compare these measurements to data taken

## Experimental Description

The microwave interferometer (Figure 1) that was developed for the 34 GHz measurements was based on the highly successful 17 GHz system already in use at the University of Michigan [1,2]. Density and spectral measurements were taken using a 34 GHz signal transmitted through the plume of the thruster orthogonal to the thruster axis.

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<sup>\*</sup> Graduate Student Research Assistant, Electrical Engineering and Computer Science

<sup>§</sup> Associate Professor, Electrical Engineering and Computer Science, member AIAA

<sup>¥</sup> Associate Professor, Aerospace Engineering, member AIAA

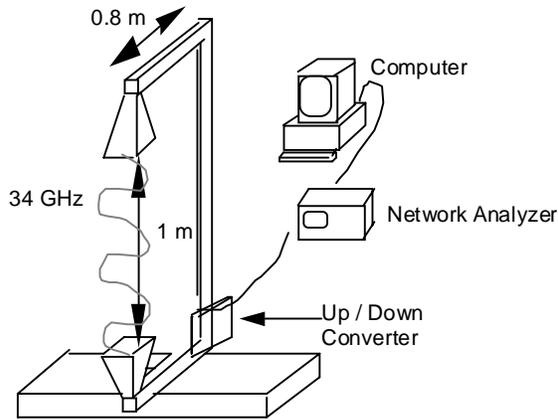


Figure 1 – 35 GHz Interferometer System

The interferometer system consisted of a computer controlled network analyzer connected to a frequency conversion circuit via flexible microwave coaxial cables and two horn antennas with a gain of 25 dB and a beamwidth of about 7 degrees. The horns were separated by 1 m. The circuit and the antennas were attached to each other using WR-15 waveguide. The measurement system used the capabilities of the Hewlett Packard 8753B network analyzer as a stable microwave source and highly sensitive receiver. By sweeping from 33 to 35 GHz the measurement used the time gating feature of the network analyzer to isolate the test signal and give more accurate measurements. A Macintosh computer using LabView through a GPIB interface controlled the network analyzer. For the spectral measurements, the network analyzer was used as the signal source and a HP spectrum analyzer was used to measure the spectrum of the transmitted signal.

Due to the difficulty of working with a 34 GHz signal and the long distance between the network analyzer outside of the chamber and the interferometer system inside, a frequency up-down converter was used (Figure 2). The frequency conversion circuit converted the 2 GHz signal from the network analyzer to 34 GHz using a 32 GHz local oscillator.

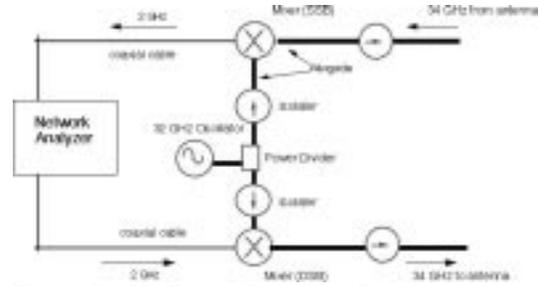


Figure 2 – Up/Down Converter Circuit.

All experimental tests were done at the University of Michigan's Plasmadynamics and Electric Propulsion Laboratory in its 6m X 9m vacuum chamber. The pressures achieved were below  $3e-7$  Torr. See [3,6] for a more complete description of the facility. Radial and axial sweeps of the plasma plume were taken by mounting the P5 thruster on a moveable platform while holding the interferometer system stationary. This was found to reduce phase noise due to vibration of the interferometer system during movement. See [6] for a more complete description of the positioning system.

In addition to 34 GHz measurements, 17 GHz measurements were also taken using the interferometer mentioned previously. The system setup was very similar to the 35 GHz setup shown in figure 1. The main difference was that the antennas were separated by 1.6 m as opposed to 1 m. A detailed description of the system can be found in [6].

The thruster used for all measurements was a 5 kW Hall Thruster developed at the University of Michigan called the P5 thruster. The thruster uses Xenon as its primary propellant. The two operating conditions used in this paper are a 300 V discharge voltage with a 5 A discharge current corresponding to a flow of 60 sscm of Xenon and a 300 V discharge voltage, with 10 A discharge current corresponding to a flow of 109 sscm of Xenon. For more information on the P5 thruster refer to [6].



Figure 3 – Picture of the P5 thruster

### Experimental Results

The results for signal phase shift at .05, .33, .5, .66 m from the thruster's exit plane at 34 GHz are shown below in Figures 4 and 5. The phase shift was measured at two thruster operating conditions; 300 Volts, 5 Amps and 300 Volts, 10 Amps.

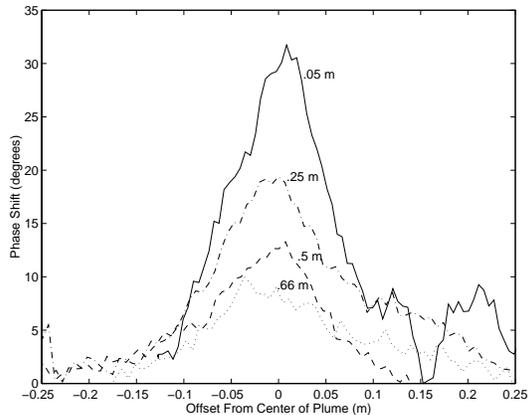


Figure 4 – Phase Shift at 35 GHz at a thruster operating condition of 300 V, 10 A.

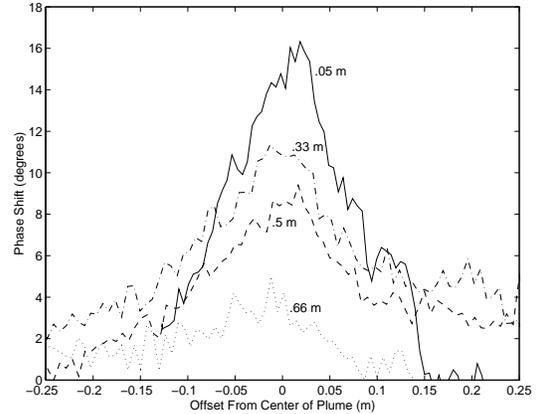


Figure 5 – Phase Shift at 35 GHz at a thruster operating condition of 300 V, 5 A.

Figures 6 and 7 show the same phase shift measurements as above taken at 17 GHz.

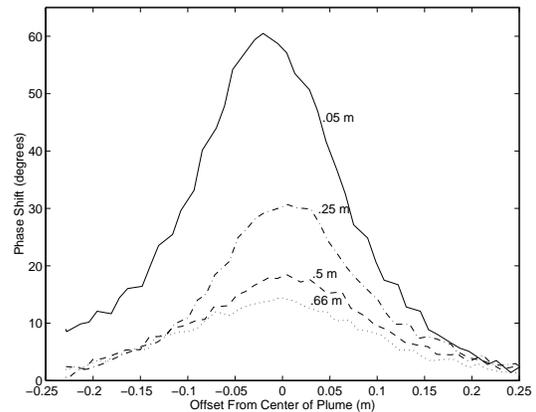


Figure 6 – Phase Shift at 17 GHz at a thruster operating condition of 300 V, 10 A.

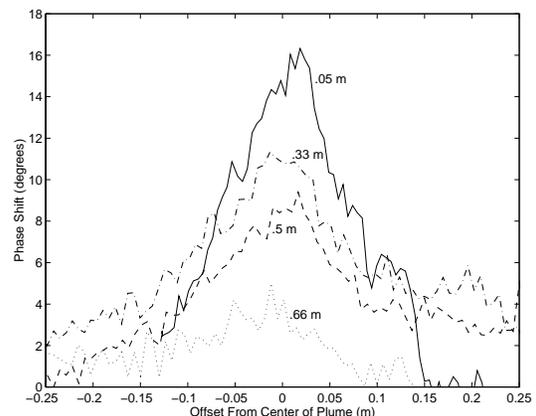


Figure 7 – Phase Shift at 17 GHz at a thruster operating condition of 300 V, 5 A.

From examining figures 4-7, it can be seen that there is well-defined region of signal phase shift within the plasma plume. As would be expected

the magnitude of the phase shift is higher at 17 GHz than at 34 GHz. It can also be seen that the width of the phase shift region is smaller in the 35 GHz measurement than the 17 GHz. The 17 GHz interferometer has a phase noise of about +/- 2 degrees and the 34 GHz system has a phase noise of about +/- 5 degrees. The 35 GHz phase oscillates more due to the fact that the wavelength is smaller and small variations in the system or background can have significant affects on the phase shift measured.

Comparing the 17 GHz measurements to similar phase measurements taken for the D-55 Anode Layer Thruster [4] shows that the phase shift region is nearly twice as wide (1 m compared to .5 m) and the phase shift is 20 degrees larger for the D-55 than the P5 at a distance of 0.5 m from the exit plane.

The signal attenuation at 34 and 17 GHz at the two different operating conditions are shown below in figures 7-10.

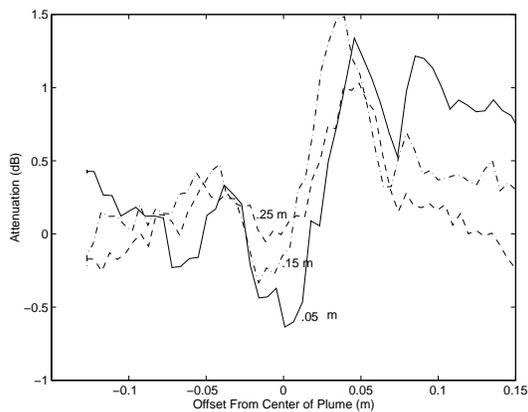


Figure 8 – Signal attenuation at .05, .15, .25 m at 34 GHz at an operating condition of 300 V, 10 A.

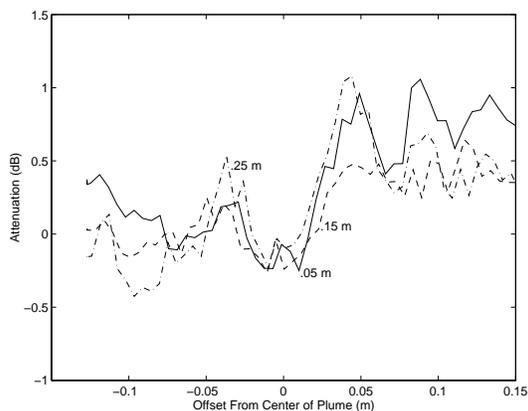


Figure 9– Signal attenuation at .05, .15, .25 m at 34 GHz at an operating condition of 300 V, 5 A.

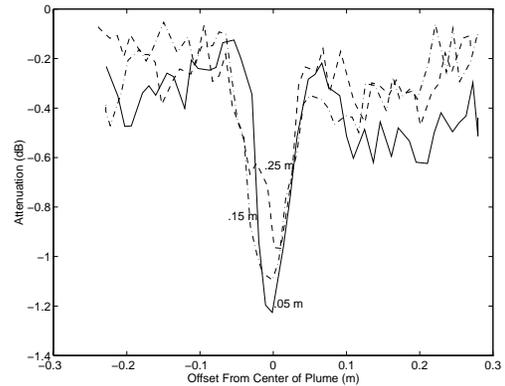


Figure 10– Signal attenuation at .05, .15, .25 m at 17 GHz at an operating condition of 300 V, 10 A.

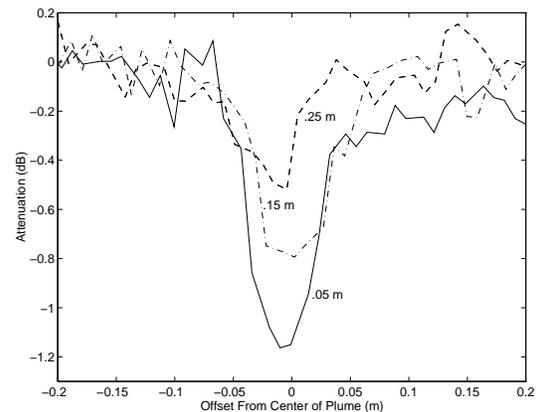


Figure 11 – Signal attenuation at .05, .15, .25 m at 17 GHz at an operating condition of 300 V, 5 A.

The attenuation for the 17 GHz case is as expected higher than the 34 GHz case (about 1.2 dB compared to .5 dB). There is also much more noise in the 34 GHz measurements and there is not as clearly defined signal drop across the center of the plume. A significant cause of extra noise in the 35 GHz system is likely due to the smaller wavelength causing more constructive and destructive interference with the received signal than in the 17 GHz system. Reflections and not the plasma caused the large spikes in the amplitude measurements at 0.05 m radially from the plume since they were still present with the thruster off.

Comparing the attenuation measurements to similar measurements for the D-55 [4], the peak attenuation for the P5 thruster is about 1 dB less than that measured for the D-55 at .25 m from

the exit plane. Also the attenuation region is narrower in the P5 thruster similar to the phase measurements. Significant attenuation (i.e. attenuation greater than the noise level) in the P5 thruster at 17 GHz is at about +/- 0.1 m from the center of plume while significant attenuation for the D-55 is at about +/- 0.2 m.

Figures 11 and 12 show the spectrums measured for a 17 and 34 GHz signal being transmitted orthogonal through the thruster's plasma plume at a distance of .05 m from the exit plane at an operating condition of 300 V, 5A.

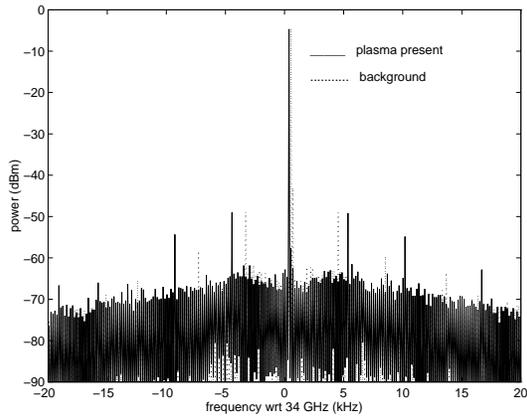


Figure 12 – 34 GHz spectrum at 300 V, 5A

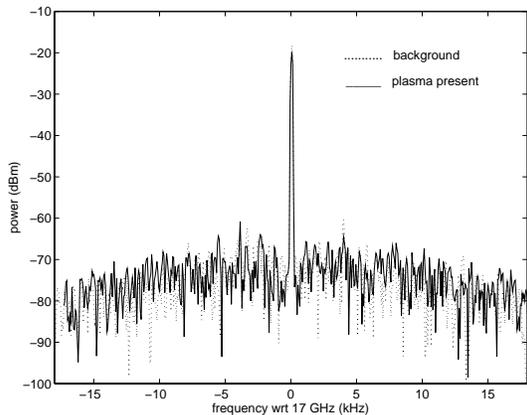


Figure 13 – 17 GHz spectrum at 300 V, 5A

As can be seen there is no significant difference between the spectrum with the plasma present and when the thruster is off for either frequency at an operating condition of 300 V, 5A.

Figure 14 shows a spectrum measurement taken at 17 GHz at an operating condition of 500 V, 10 A [7]. Harmonics can be clearly seen at 12 kHz about 33 dB down from the main signal.

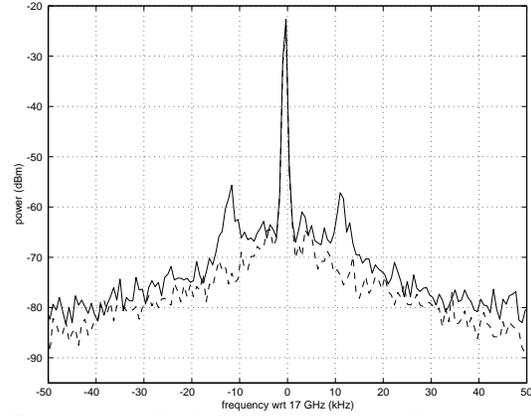


Figure 14 – 17 GHz spectrum at 500 V, 10 A

This demonstrates the dependence of plasma oscillations in this frequency range on discharge voltage and current. According to the above results, a higher discharge voltage gives a stronger oscillation. More research is planned to correlate plasma oscillations to thruster operating conditions and also to see what affect these oscillations have on thruster performance.

## Electron Density Calculations

To measure the electron density of a plasma microwave interferometry uses the integrated phase shift of an electromagnetic wave passing through the plasma at a sweep of radial values as measured previously. Electron density profiles are then calculated based on these measured phase shifts using the following Abel inversion.

$$n(r) = \frac{\lambda n_c}{\pi^2} \int_r^R \frac{\partial \theta(x)/\partial x}{\sqrt{x^2 - r^2}} dx \quad (1)$$

The implementation of the integral is not straightforward due to the derivative of the data and the pole at the integral. The method used to solve this integral was to fit a polynomial to the phase shift data. This provided an analytical solution, which eliminated the problem with the derivative or pole [8]. In order for microwave interferometry to be a valid technique of calculating electron density, the following condition must hold.

$$\lambda \ll \frac{n_c}{\partial n_e / \partial r} \quad (2)$$

Also, the plasma must be stable over the measurement duration, which is approximately 5

minutes. Both of these conditions are satisfied in the P5 thruster.

The bumps in number density at larger radial distances are the result of phase noise in the measurements, which is more significant at the edges of the electron density profile.

The calculated electron density profiles for 35 GHz and 17 GHz are shown below for the two operating conditions.

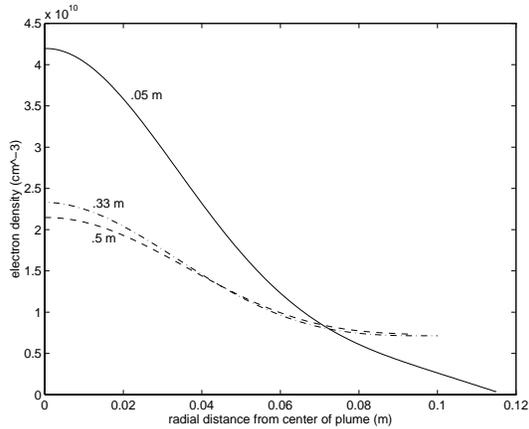


Figure 13 – Electron density at 34 GHz at 300 V, 10 A.

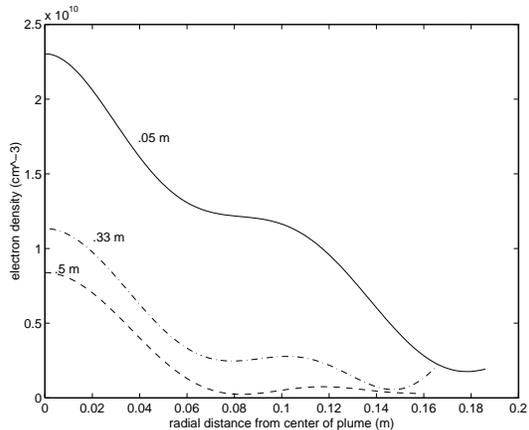


Figure 14 – Electron density at 34 GHz at 300 V, 5 A.

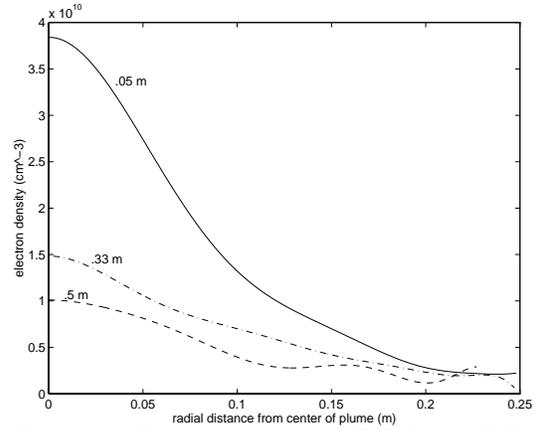


Figure 15 – Electron density at 17 GHz at 300 V, 10 A.

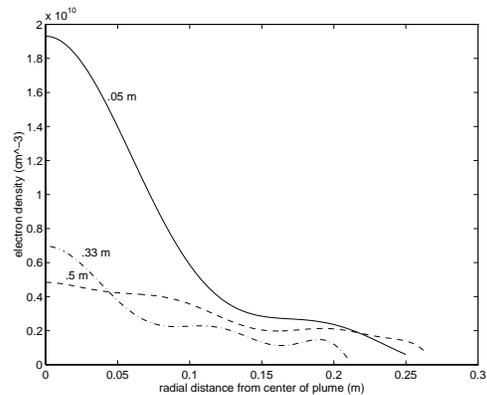


Figure 16 – Electron density at 17 GHz at 300 V, 5 A.

From looking at the plots, it can be seen that the peak electron density at 300 Volts and 5 Amps is about  $2.3 \times 10^{10} \text{ cm}^{-3}$  for the 35 GHz case and  $1.9 \times 10^{10} \text{ cm}^{-3}$  for the 17 GHz case at .05 m. For the 10 Amp case, the peak electron density is about  $3.8 \times 10^{10} \text{ cm}^{-3}$  and  $4.2 \times 10^{10} \text{ cm}^{-3}$  for 17 and 35 GHz, respectively. The electron densities are nearly a factor of two higher for the 10 Amp case compared to the 5 Amp, which would be expected since nearly twice as much Xenon is being fed into the thruster.

Figure 17 shows a comparison of the electron density profiles calculated for the 17 and 34 GHz signals.

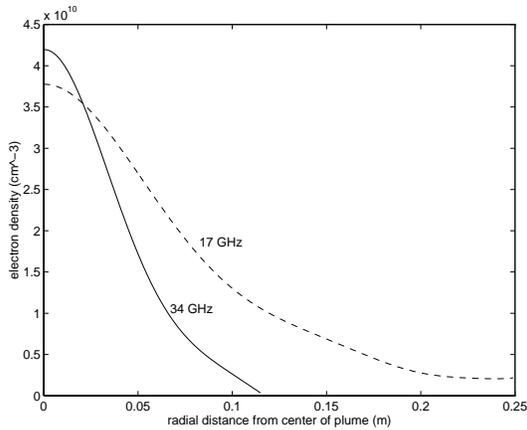


Figure 17 – Comparison between 17 and 34 GHz density profiles at .05 m from the exit plane at 300 V, 10 A.

As is shown in the figure, the width of the electron density distribution of the thruster was calculated to be nearly a factor of two larger for 17 GHz case than the 34 GHz. This is because the spot size of the beam on the plasma for the 34 GHz system is smaller than that of the 17 GHz system. The reason being that the 35 GHz horn antennas could be placed closer together due their far-field zone being smaller. Therefore, since a smaller cross-section of the plasma was seen with each measurement the overall resolution of the density measurements was better with the 35 GHz system giving a more accurate prediction of the width of the electron density profile of the thruster. It should also be noted that the peak density for the 35 GHz is consistently slightly higher than those for the 17 GHz measurements, though the measurements are relatively close. This demonstrates the repeatability of microwave interferometry since these measurements were taken on different days.

Comparing these electron densities to densities measured using Langmuir probes on the P5 under similar conditions [3] shows that the microwave interferometer measurements are lower by about a factor of 10. At 300 V, 5 A at a distance of 0.5 m, the Langmuir probes measured a density of about  $8 \times 10^{10} \text{ cm}^{-3}$  compared to  $8 \times 10^9 \text{ cm}^{-3}$  measured using interferometry. Similar discrepancies, though, were found with data from measurement of the D-55 thruster [4]. The shape of the profile is consistent between both measurement techniques.

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