

Hall Thruster Electron Motion Characterization Based on Internal Probe Measurements

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Jesse A. Linnell¹ and Alec D. Gallimore²
University of Michigan, Ann Arbor, MI 48109 USA

Abstract: In this study, electron behavior inside the discharge channel of the NASA-173Mv1 Hall thruster is characterized using a combination of internal probe measurements and the electron fluid equations of motion. Two-dimensional mappings of the electron Hall parameter, total electron collision frequency, electron mobility, and azimuthal electron current are shown. The experimental Hall parameter in the near anode region varies between 1 and 100, climbs to a value of approximately 400 at the thruster exit, and decreases to less than the Bohm value ($\Omega_e=16$) in the downstream region. The electron collision frequency ranges between 10^7 and 10^9 Hz in the interrogation zone, and reaches a minimum of 10^6 Hz at the thruster exit. Axial and radial electron mobility is found to be strongly dependant on magnetic field strength and orientation. The electron mobility ranges mostly between 10^2 to 10^4 and peaks around 2×10^5 C-s-kg⁻¹. Axial electron mobility peaks along axially directed magnetic field lines and radial electron mobility peaks at the thruster exit. The azimuthal current is modeled using a combination of $E \times B$ and diamagnetic current. The peak diamagnetic current is found to be 20-25% of the peak $E \times B$ current. This result suggests that diamagnetic current is significant and should be included in Hall thruster models.

Nomenclature

A_{ch}	=	discharge channel cross-sectional area
B	=	magnetic flux density
D	=	electron diffusivity
E	=	electric field
e	=	elementary charge
$\hat{e}$	=	unit vector
I_D	=	discharge current
J_{ez}	=	electron current density to the anode
k_B	=	Boltzmann constant
m_e	=	electron mass
M_i	=	ion mass
n_e	=	electron number density
n_i	=	ion number density
$\overline{\overline{T}}$	=	transformation tensor

¹ Technical Staff, Advanced System Concepts, MIT Lincoln Laboratory, jlinnell@ll.mit.edu, Research performed as a doctoral candidate at the Plasmadynamics and Electric Propulsion Laboratory at the University of Michigan.

² Arthur F. Thurnau Professor of Aerospace Engineering and Director of the Plasmadynamics and Electric Propulsion Laboratory, University of Michigan, alec.gallimore@umich.edu.

T_e	=	electron temperature
u_i	=	ion velocity
V_e	=	electron velocity
V_p	=	plasma potential
x	=	spatial variable
μ	=	electron mobility
$\nu_{e,tot}$	=	total electron collision frequency
ϕ	=	magnetic field line angle of rotation
Ω_e	=	electron Hall parameter
ω_c	=	electron cyclotron frequency

Subscripts

k	=	azimuthal direction relative to B-field coordinates
r	=	radial direction
z	=	axial direction
θ	=	azimuthal direction relative to thruster coordinates
$\perp$	=	cross-field direction
$\parallel$	=	parallel-field direction

I. Introduction

Electron motion internal to a Hall thruster is a poorly characterized and a poorly understood process. To assist in the understanding of the electron dynamics, internal probe measurements are used in combination with a fluid description of the electron motion. The results of this analysis create two-dimensional mappings of the discharge channel. Properties studied in this analysis are electron Hall parameter, total electron collision frequency, electron mobility, and azimuthal electron current.

It is the goal of this analysis to improve the understanding of electron behavior inside Hall thrusters. These results should assist researchers by helping to further develop electron mobility models. It is also desired that this analysis will create a baseline for experimentation and analysis in which future studies can build from.

Previous efforts have studied the internal electron behavior through experimentation and modeling. Meezan et al.,¹ Haas,² and Reid³ have conducted experiments to characterize Hall thruster electron properties. The uniqueness of this analysis is in the two-dimensional nature of these results. Many modeling efforts have focused on internal Hall thruster electron behavior because of its great importance to the modeling effort.⁴⁻¹⁰ Several of these studies will be discussed in the text below.

II. Hall Thruster Operation and Measurements

For this investigation, experiments were conducted using the NASA-173Mv1 Hall thruster^{11,12} at the University of Michigan's Plasmadynamics and Electric Propulsion Laboratory. The measurements were taken in the Large Vacuum Test Facility, which is a 6 m in diameter and 9 m long cylindrical stainless steel tank. A series of emissive and single Langmuir probes were used to measure electron temperature, plasma potential, and ion number density throughout the discharge channel.¹³⁻¹⁵ The details and results of these experiments have been reported previously and will not be discussed here. These measurements yield plasma properties that are used in the following analysis. The electric field is calculated by taking the derivative of the plasma potential and electron number density is estimated by assuming quasineutrality. The magnetic fields were modeled using the 3D magnetostatic solver Magnet 6.0 by Infolytica. The discharge channel dimensions for the NASA-173Mv1 are 25.4 mm in width and 38 mm in depth.

Table 1. Operation Points for the NASA-173Mv1

Operation Point	A	B
Discharge Voltage, V	300	500
Cathode Potential, V	-10.9	-11.7
Discharge Current, A	8.95	9.27
Anode Flow, mg/s	10.00	10.00
Cathode Flow, mg/s	1.00	1.00
Inner Coil Current, A	1.88	2.90
Outer Coil Current, A	2.21	2.87
Trim Coil Current, A	-0.51	-0.87
Thrust, mN	180	247
Anode Efficiency, %	61.2	66.1

The thruster was operated at 300 and 500 V with 10 mg/s of xenon anode flow. The specifics of each operating condition are given in Table 1.

III. Electron Motion Analysis

This section will discuss the theory and method that is used to study the electron dynamics inside the Hall thruster. Along with presenting the electron equations of motion, this section will also discuss the analysis process for the calculation of the electron Hall parameter, the total electron collision frequency, the electron mobility, and the azimuthal electron current. Lastly, this section will discuss the assumption used to approximate of electron current toward the anode.

A. Electron Fluid Equations of Motion

For this analysis, a fluid description of the electron motion is used since the bulk plasma trends are expected to be much more important than the single particle effects. By taking the electron fluid momentum equation and assuming the electron motion is in steady state and that the inertial term can be neglected, one can arrive at the Eqs. 1-3. For this analysis, the azimuthal component of magnetic field is considered negligible, the plasma is assumed to be axisymmetric, and electrons are assumed to be isothermal along the magnetic field lines. In order to simplify the equations of motion, the coordinate system is set relative to the magnetic field lines. For clarity, the $\perp$, $\parallel$, and k subscripts are used to represent the perpendicular-field, parallel-field, and azimuthal directions, respectively. Equation 1 gives the electron velocity parallel to the magnetic field lines, Eq. 2 is the electron velocity perpendicular to the magnetic field lines, and Eq. 3 is the azimuthal electron velocity.

$$V_{e\parallel} = -\mu E_{\parallel} - D \frac{\nabla_{\parallel} n_e}{n_e} - D \frac{\nabla_{\parallel} B}{B} \quad (1)$$

$$V_{e\perp} = -\mu_{\perp} E_{\perp} - \frac{D_{\perp}}{n_e} \frac{\partial n_e}{\partial x_{\perp}} - \frac{D_{\perp}}{T_e} \frac{\partial T_e}{\partial x_{\perp}} \quad (2)$$

$$V_{ek} = \frac{\Omega_e^2}{1 + \Omega_e^2} \left(\frac{E_{\perp}}{B} + \frac{k_B T_e}{e B n_e} \frac{\partial n_e}{\partial x_{\perp}} + \frac{k_B}{e B} \frac{\partial T_e}{\partial x_{\perp}} \right) \quad (3)$$

To complete the analysis several additional terms are given below. The electron Hall parameter and cyclotron frequency are given and Eqs. 4 and 5, respectively. The definitions of electron mobility and cross-field electron mobility appear in Eqs. 6 and 7. The definition of diffusivity and cross-field diffusivity appear in Eqs. 8 and 9.

$$\Omega_e = \frac{\omega_c}{v_{e,tot}} = -\frac{V_{ek}}{V_{e\perp}} \quad (4)$$

$$\omega_c = \frac{eB}{m_e} \quad (5)$$

$$\mu = \frac{e}{v_{e,tot} m_e} \quad (6)$$

$$\mu_{\perp} = \mu \left(\frac{1}{1 + \Omega_e^2} \right) \quad (7)$$

$$D = \frac{k_B T_e}{\nu_{e,tot} m_e} \quad (8)$$

$$D_{\perp} = D \left(\frac{1}{1 + \Omega_e^2} \right) \quad (9)$$

Along the field lines, the electrons move relatively freely due to a random diffusion process and a balance between the electric field, the pressure forces, and the magnetic mirror force.^{16,17} With exception to the magnetic mirror force term, Eq. 1 is the classic form of the equation for unmagnetized electrons. As shown by these equations, the electron mobility and diffusivity are greatly reduced across the magnetic field lines. This reduction in mobility is related to the Hall parameter given in Eq. 4. Equation 3 shows the electron velocity in the azimuthal direction. On the right hand side of the equation, the first term is the $E \times B$ drift and the second two terms are the diamagnetic drift. In Eq. 3, the Hall parameter coefficient becomes negligible for Hall parameters greater than 10.

B. Hall Parameter

The electron Hall parameter (Eq. 4) is one of the major parameters describing the electron motion inside a Hall thruster. By definition, the Hall parameter is a ratio between the electron cyclotron frequency and the total electron collision frequency. Physically, the Hall parameter characterizes the number of azimuthal orbits that an electron completes before undergoing a particle collision. The electron-particle collision results in cross-field migration and eventually the loss of electrons to the anode. It can be shown by dividing Eq. 3 by Eq. 2 that the Hall parameter is a ratio between the azimuthal and perpendicular electron velocities.

In the following analyses, where the cyclotron radius larger is than 1/15 of the discharge channel width, the electrons are considered to be unmagnetized. In unmagnetized regions the azimuthal current is set to zero and the Hall parameter is set to one. These values are selected to prevent plasma properties for going to infinity. The regions of unmagnetized plasma are excluded from the Hall parameter scatter plot. These regions can also be identified in the contour plots by the smooth contours and proximity to the anode.

C. Electron Collision Frequency

Important to this discussion is the total electron collision frequency, which is given the symbol $\nu_{e,tot}$. In the absence of collisions, the electrons are restricted from crossing the magnetic field lines. When an electron experiences a momentum exchange collision, it undergoes a random-walk process with step sizes on the order of a Larmor radius. When an electron collides with a heavy particle, the cross-field transport is called classical mobility. However, classical type collisions under predict the electron collision rate observed within Hall thrusters. To account for this discrepancy, a combination of three electron transport mechanisms is commonly used. These mobility types are classical mobility and two forms of anomalous electron mobility, which are near-wall conductivity and Bohm mobility. The resulting equation for total electron collision rate is: $\nu_{e,tot} = \nu_{Classical} + \nu_{Bohm} + \nu_{Wall}$.

Near-wall conductivity theory proposes that electron collisions with the walls enhance electron cross-field mobility and in this way the walls act like a macro-particle.¹⁸ Another explanation for the anomalous behavior is Bohm mobility, $\nu_{Bohm} = \alpha_B \omega_c$. Bohm mobility stems from the turbulent fluctuations in the electric field and plasma density. Evidence of this anomalous Bohm mobility has been observed experimentally¹ and is necessarily imposed in order to match computational models to experimental results. Typically the coefficient (α_B) used to model the Bohm collision frequency is 1/16. However, in many modeling efforts anomalous mobility is only applied in specific regions of the Hall thruster model. Typical values of approximately 1/100 are found to be appropriate.^{4,6-8,19}

The experimental results presented below are not able to separate the different transport mechanisms and experimental results are a combination of all mobility mechanisms. Unfortunately, these results are not well suited to study near-wall conductivity since the closest measurements are 2 mm from the wall.

D. Electron Mobility

With the experimental collision frequency and the Hall parameter it is possible to calculate the mobility parallel and perpendicular to the magnetic field lines (Eqs. 6 and 7). Equations 10-12 are used to transform the electron mobility from magnetic field coordinates to thruster coordinates. In these equations, $\bar{\bar{T}}$ is the transformation tensor, ϕ is the angle of rotation of the magnetic field line, and $\hat{e}$ is a unit vector.

$$\overline{\overline{\mu}}_{Thruster} = \begin{bmatrix} \mu_z & \mu_{zr} & 0 \\ \mu_{rz} & \mu_r & 0 \\ 0 & 0 & \mu_g \end{bmatrix} = \overline{\overline{T}}^T \begin{bmatrix} \mu_{\perp} & 0 & 0 \\ 0 & \mu_{\parallel} & 0 \\ 0 & 0 & \mu_k \end{bmatrix} \overline{\overline{T}} \quad (10)$$

$$\begin{pmatrix} \hat{e}_{\parallel} \\ \hat{e}_{\perp} \\ \hat{e}_k \end{pmatrix} = \overline{\overline{T}} \begin{pmatrix} \hat{e}_z \\ \hat{e}_r \\ \hat{e}_{\theta} \end{pmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \hat{e}_z \\ \hat{e}_r \\ \hat{e}_{\theta} \end{pmatrix} \quad (11)$$

$$\tan \phi = \frac{B_z}{B_r} \quad (12)$$

E. Azimuthal Electron Currents

The azimuthal electron drifts can be calculated directly from Eq. 3. Equation 13 is used to calculate the azimuthal electron current density in the discharge channel. Electron drifts are converted into electron currents by assuming quasineutrality and multiplying the drift velocities by the ion number density and the elementary charge. In the results section, the $E \times B$, the diamagnetic, and total azimuthal current density are given to show their relative importance.

$$J_e = en_e V_{\theta} \quad (13)$$

This analysis uses a fluid approach to studying the electron motion and for this reason several particle drifts are ignored. Electron drifts can behave differently depending if a fluid or particle description is used. For example, the diamagnetic drift is caused by gradients in plasma pressure and for this reason it does not exist in the particle sense. Several particle drifts including nonuniform electric field drift, curvature drift, and grad-B drift have been evaluated elsewhere and are shown to be less than a few percent of the $E \times B$ drift.²⁰ King also studied the electron motion inside the NASA-173Mv1 using a guiding center approach.²¹

F. Anode Current Assumption

Given Eqs. 1 to 9, the fluid description of the electron motion is complete. Closure of these equations is accomplished by using experimental data and one assumption. This section will discuss the estimation of the electron current to the anode.

The axial electron current density toward the anode (J_{ez}) is found by using Eq. 14. It is assumed that the electron current density is uniform at each axial location in the discharge channel. The ion current density is calculated based on measured ion number densities, and the axial ion velocity is calculated from the plasma potential measurements and using a one-dimensional energy balance in Eq. 15. The axial ion velocity at 10 mm ($u_{iz}|_{10mm}$) is assumed a half Maxwellian with an ion temperature of 0.1 eV. The electron current is calculated by the difference of the discharge current and ion current. This assumption is similar to the analysis conducted by Meezan et al.¹ In these equations, all of the variables have their usual meanings.

$$J_{ez}(z) = \frac{I_D - \int_{A_{ch}} en_i u_{iz}|_z dA}{A_{ch}} \quad (14)$$

$$u_{iz}(z)^2 = \sum_{i=0}^{i=z} \frac{2e}{M_i} (V_{p,i-1} - V_{p,i}) + (u_{iz,10mm})^2 \quad (15)$$

The axial electron current toward the anode is shown in Fig. 1. The estimation of electron current in this way is common,¹ however it is best to proceed with caution. Downstream of the chamber exit, the ion number density

begins to drop because of ion beam divergence. Since the ion velocity is assumed to be purely axial in Eqs. 14 and 15, the ion current is under predicted. This results in a rise in axial electron current downstream of the thruster exit, which can be seen in Fig. 1. This has implications for inaccuracies in the downstream region. This effect will tend to under predict the experimental Hall parameter and cross-field electron mobility and over predicting the electron collision rate and electron mobility in the downstream region. In the downstream region, this will result in inaccuracies around 2-4 times the correct value. Fortunately, the upstream and near discharge channel regions are of greatest interest and are expected to be much more accurate.

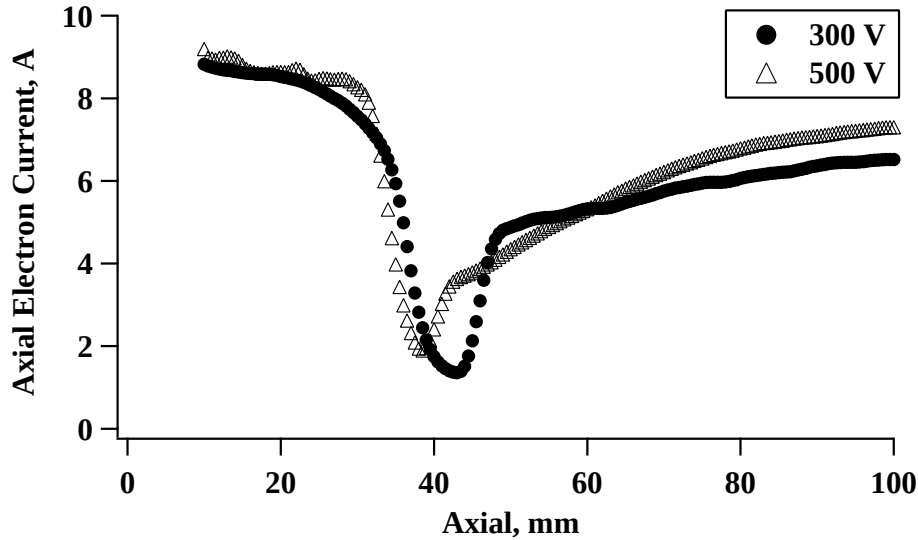


Figure 1. Axial Electron Current Toward the Anode

IV. Results

A. Hall Parameter

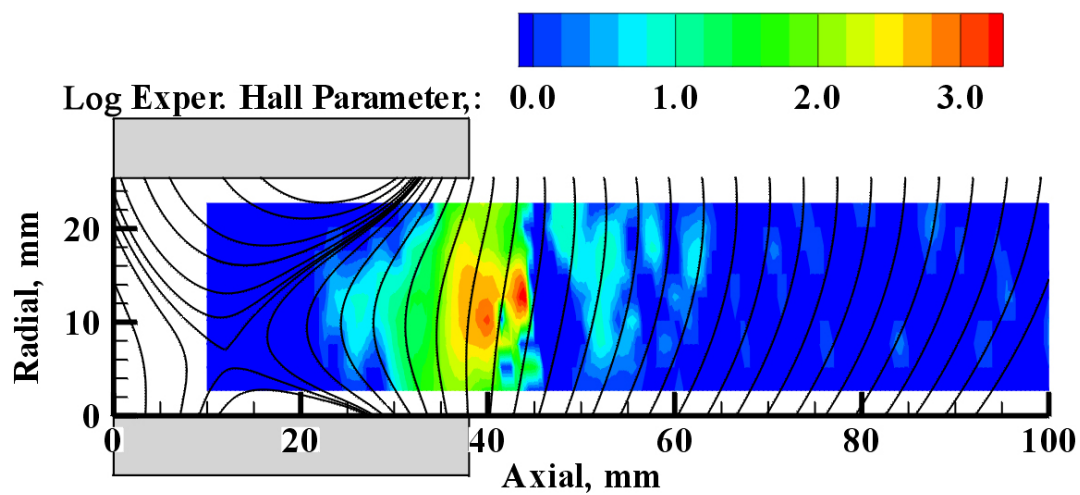
Contour plots of the experimental Hall parameter for the 300 and 500-V cases are shown in Fig. 2. Centerline values of the experimental Hall parameter appear in Fig. 3. The Hall parameter is low in the near anode and downstream region and peaks near the exit of the thruster. This location corresponds to the region with the peak magnetic field strength and the acceleration zone.

The Hall parameter in the near anode region is between 1 and 10 for the 300-V case and between 10 and 100 for the 500-V case. The larger Hall parameter values in the 500-V case are caused by the larger azimuthal drift in the near anode region. The increased azimuthal drift is largely caused by the diamagnetic current. Results for the diamagnetic current are shown in Section IV, D. Other researchers have studied the Hall parameter with varying conclusions. Meezan et al.¹ found the Hall parameter to be close to classical near the anode where Haas² and Choueiri⁵ found the near anode region to be closer to Bohm mobility.

Near the thruster exit, the Hall parameter peaks to a value of approximately 400 for both cases. These calculated Hall parameter values are consistent with work done by Haas,² Choueiri,⁵ Reid,³ and Meezan et al.¹

In the downstream region, both cases match well with Hall parameters between 1 and 10. As mentioned previously, the experimental Hall parameter is expected to be under predicted by a factor of 2-4. With this error in mind, the experimental Hall parameter is slightly smaller than the Hall parameter predicted by Bohm mobility alone. This suggests that the effects of classical mobility may still be important when modeling this region. Meezan et al.¹ found a Bohm-like mobility to be appropriate at and downstream of the peak magnetic field region although, this approximation fails to account for the high Hall parameter region at the exit of the thruster. Others also suggest using a Bohm mobility in the downstream region.^{5,9} Haas saw that classical mobility is closer to predicting the Hall parameter downstream of the acceleration zone.

a)



b)

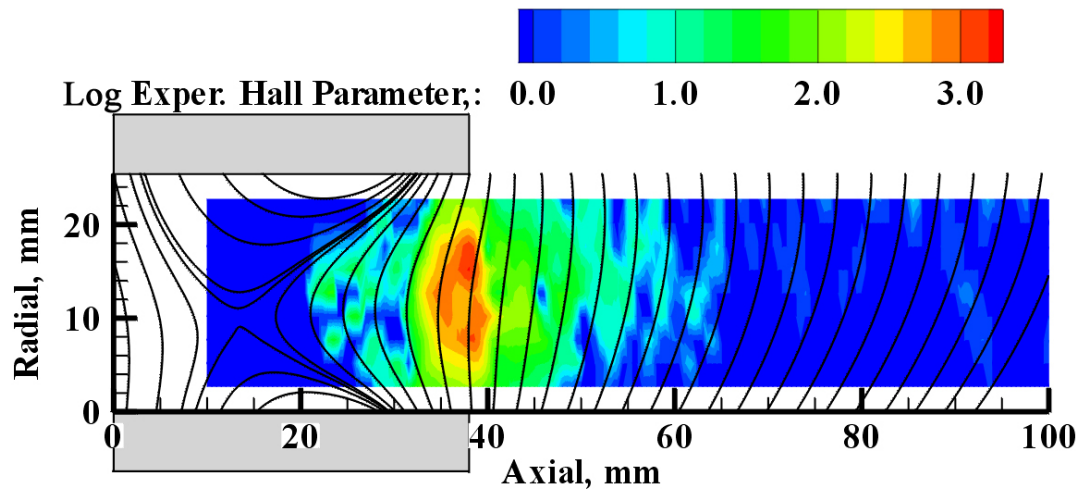


Figure 2. Experimental Hall Parameter for a) 300 and b) 500-V Cases

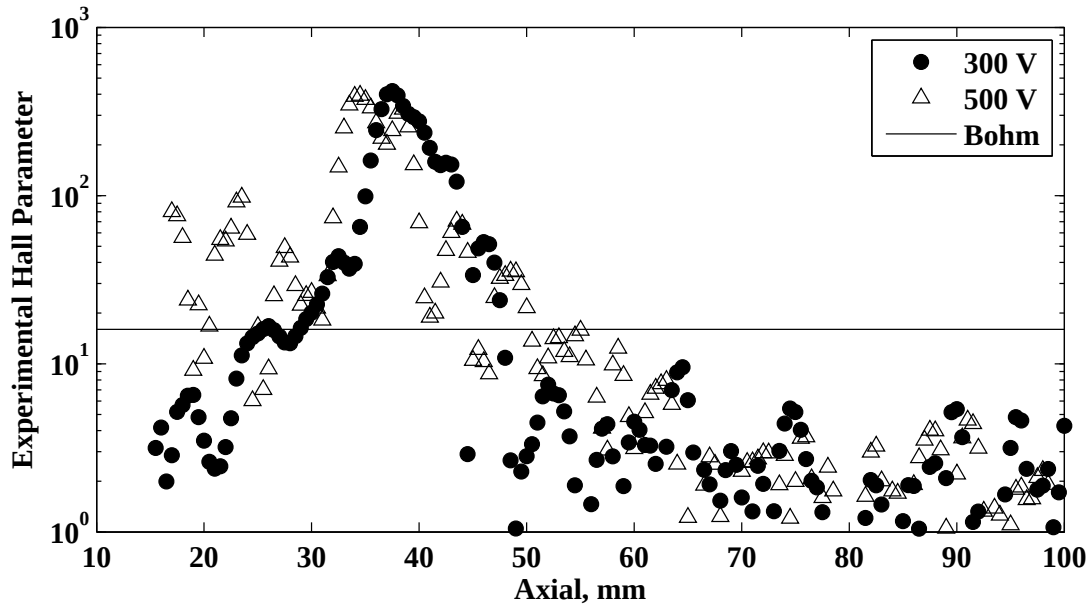


Figure 3. Centerline Experimental Hall Parameter for 300 and 500-V Cases

B. Electron Collision Frequency

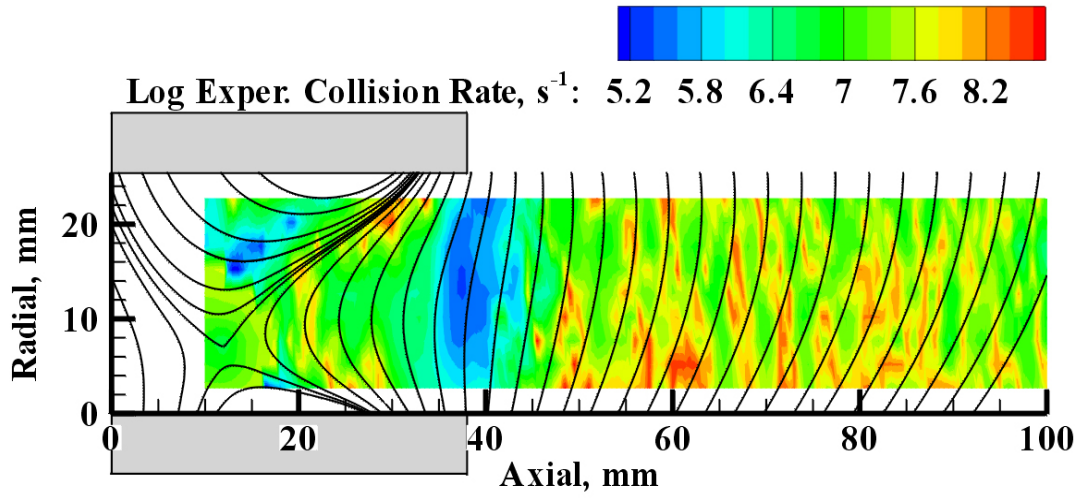
Two-dimensional mapping of the total electron collision frequency is shown in Fig. 4 and centerline collision frequency is given in Fig. 5. The total electron collision frequency starts around 10^7 Hz in the near anode region and drops to 10^6 or below near the exit of the thruster. Electron collision frequency quickly rises in the downstream region to a value of 10^9 and slowly drops to between 10^7 and 10^8 . These ranges of collision frequency have been observed in modeling efforts.^{6,19}

In the near anode region, the collision frequency is relatively high and the electrons flow more freely toward the anode. This is particularly true in regions where the magnetic field lines run axially. These locations predict very low collision rates because few collisions are necessary to account for the axial flow of electrons assumed in Section III.F. In the regions where the magnetic field is very low, the plasma is considered unmagnetized the Hall parameter is assumed to be equal to 1. This can be seen by the smooth contours, which correspond to the zero magnetic field regions in the discharge channel.

Near the thruster exit the electron collision rate decreases. Agreement between the Bohm collision rate and the experimental results is very poor in this region. This illustrates the weakness of the Bohm mobility assumption. Clearly a more robust description of Bohm-like mobility is desired to improve the understanding of anomalous electron mobility in the acceleration zone.

Downstream of the acceleration zone, the experimental mobility is larger than the Bohm mobility. Even with the expected inaccuracies in the downstream region, the Bohm mobility falls slightly short of predicting the electron collision rate. This again suggests the importance of including classical effects in downstream region. The shape of the Bohm collision frequency agrees nicely in this region.

a)



b)

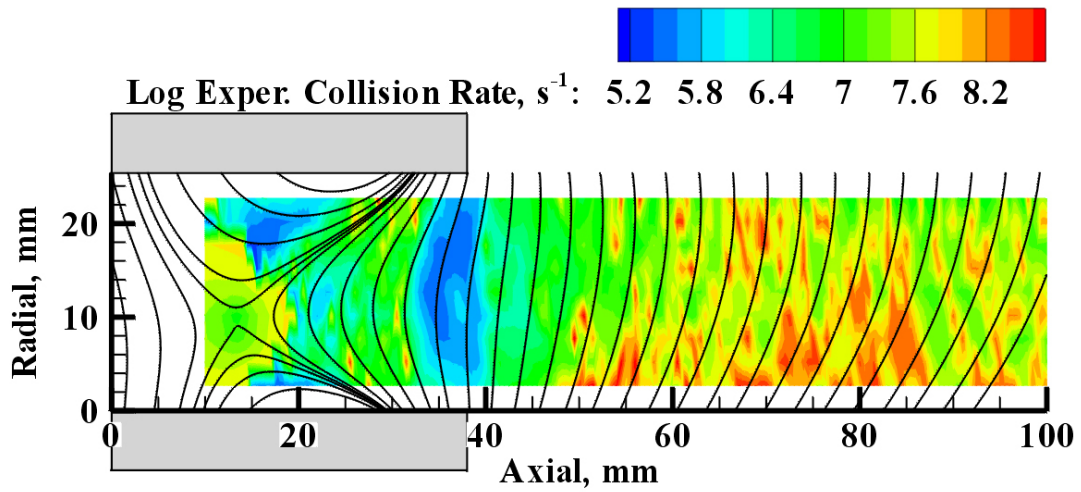


Figure 4. Experimental Electron Collision Rate for a) 300 and b) 500-V Cases

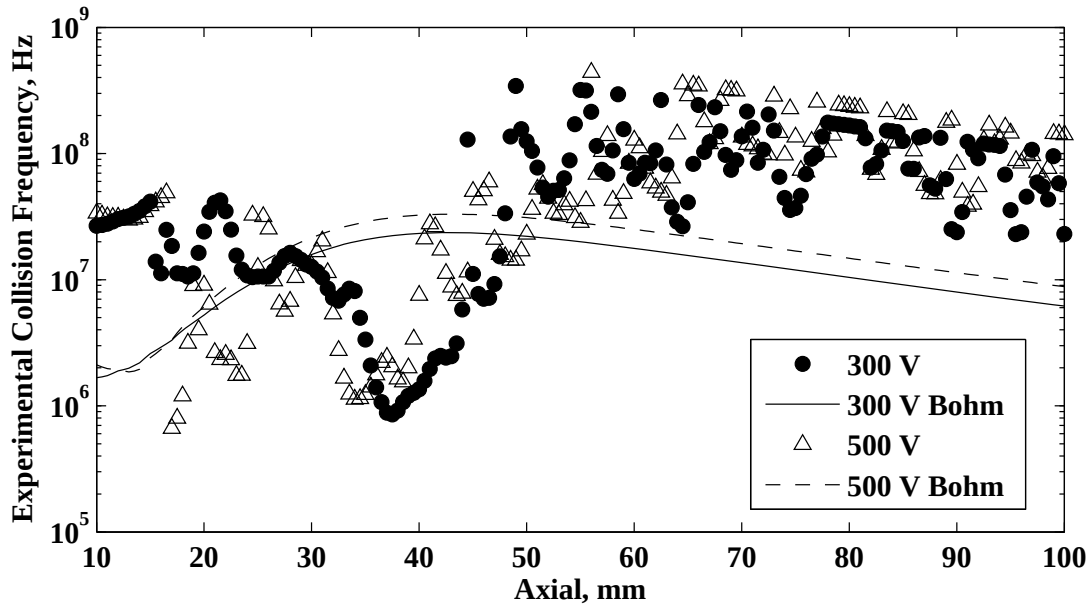


Figure 5. Centerline Experimental Electron Collision Rate for 300 and 500-V Cases

C. Electron Mobility

Contour plots for the experimental axial, radial, and R-Z electron mobility are shown in Figs. 6 and 7 for the 300 and 500-V cases, respectively. Centerline profiles for these cases are shown in Fig. 8. The axial mobility is enhanced in the regions of axially directed magnetic fields and peaks around 2×10^5 C-s-kg⁻¹. This is because the electron current is assumed to be purely axial and the cross field electron current is very low. The radial mobility peaks at the thruster exit and reaches a value of approximately 2×10^5 C-s-kg⁻¹. The R-Z mobility peaks in magnitude around -4×10^4 C-s-kg⁻¹. Results for both operation points are consistent.

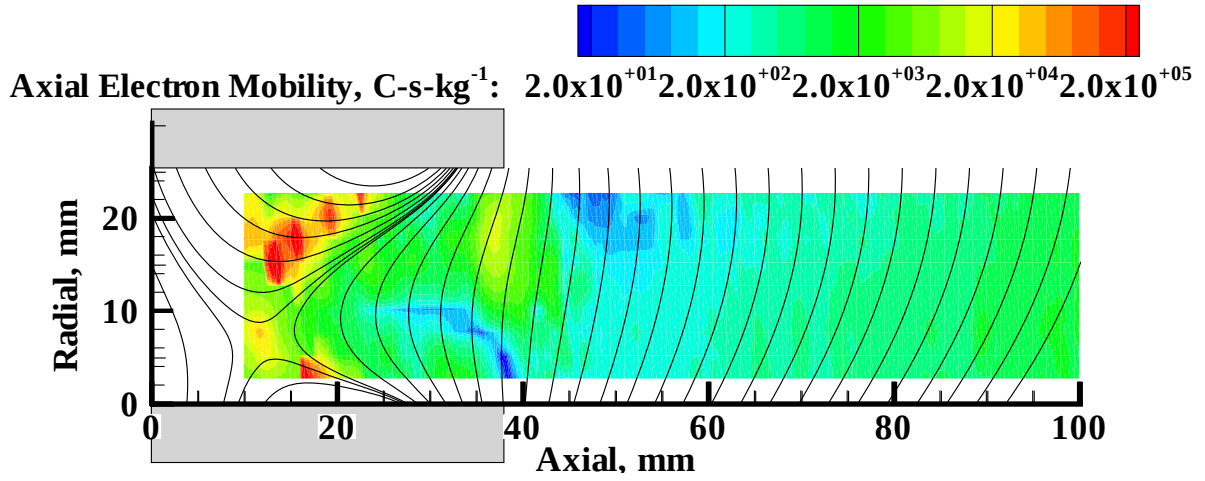
Near the anode, along the centerline the axial electron mobility decrease from a peak of 10^4 - 10^5 C-s-kg⁻¹ to a value of between 10^2 - 10^3 C-s-kg⁻¹. This is due to the increasing magnetic field strength in this region. The axial electron mobility then peaks to a few thousand and then falls after the thruster exit. In the downstream region the axial electron mobility slowly increases.

The radial electron mobility along the centerline is relatively flat in the near anode region around a value of 10^4 and 10^5 C-s-kg⁻¹ for the 300 and 500-V cases, respectively. The radial electron mobility then sharply increases near the thruster exit and drops outside the thruster. The peak radial electron mobility region occurs where electron temperature is high and the magnetic field lines are mostly radial. This region of high radial electron mobility indicates where the electron-wall collision rate is highest.

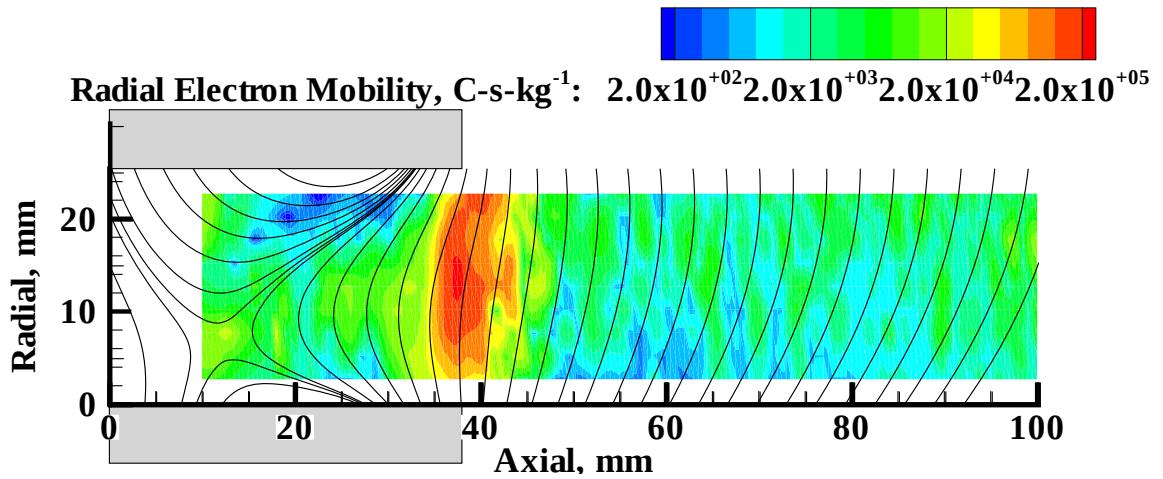
D. Azimuthal Electron Currents

Contour plots of the azimuthal electron current densities appear in Figs. 9 and 10 for the 300 and 500-V cases, respectively. Included in the figures are the $E \times B$, the diamagnetic, and the total (or the summed) current densities. The $E \times B$ current peaks around 4×10^5 and 6×10^5 A/m² for the 300 and 500-V cases, respectively. The peak diamagnetic current is between 20-25% of maximum $E \times B$ current for both cases. While the diamagnetic current is a second-order effect, it is still significant. Although the Hall current normally only refers to the $E \times B$ current, these results show the importance of the diamagnetic current. However, it is not clear the most appropriate method of handling the diamagnetic drift. After all, the diamagnetic drift results from gradients plasma pressure instead of the more traditional single particle drift.

a)



b)



c)

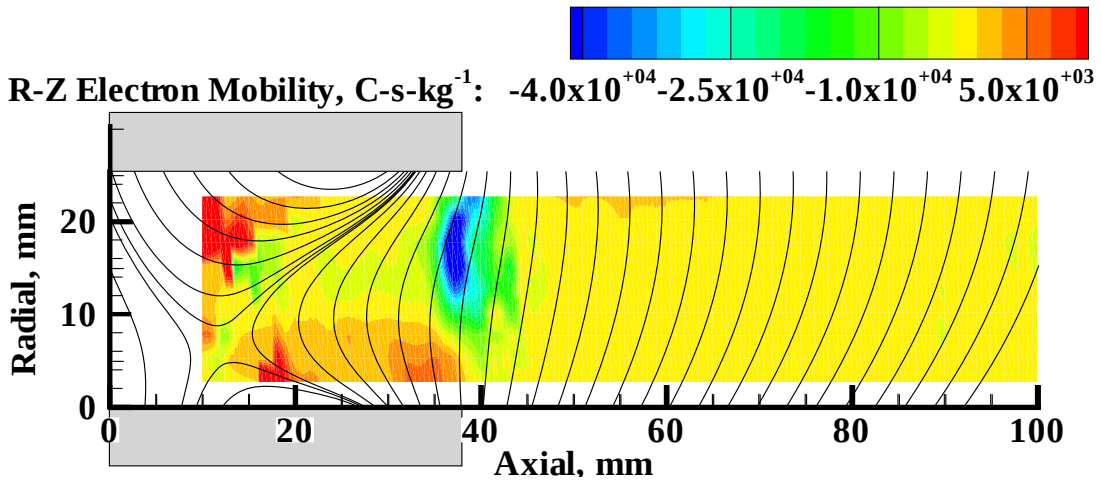
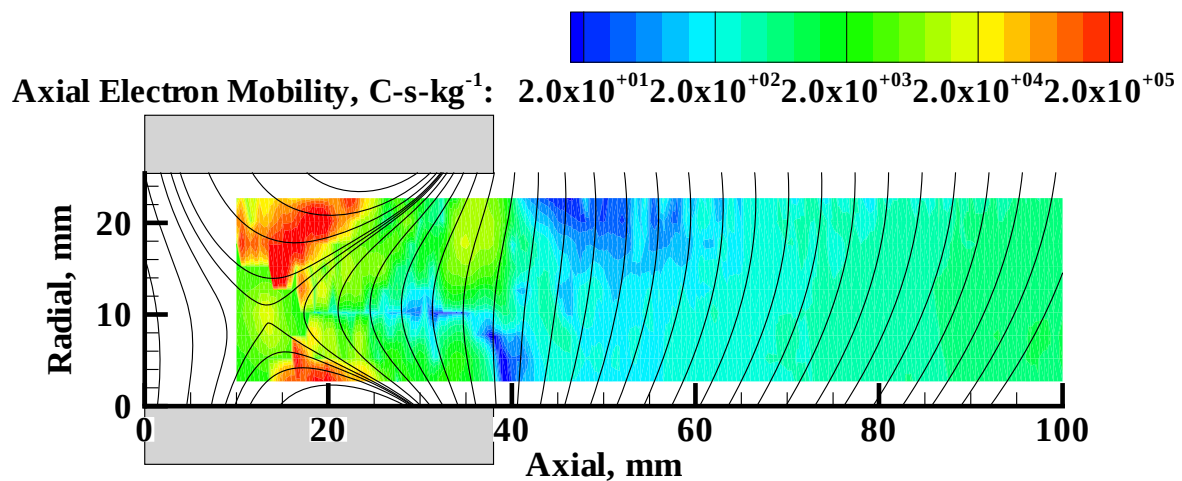
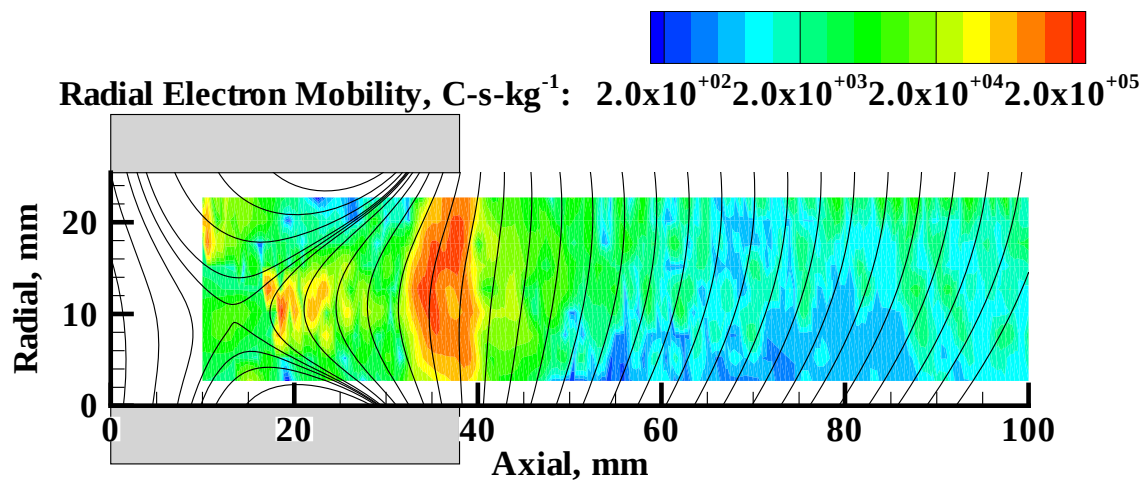


Figure 6. Experimental Electron Mobility for the 300-V Case, a) Axial, b) Radial, and c) R-Z Electron Mobility

a)



b)



c)

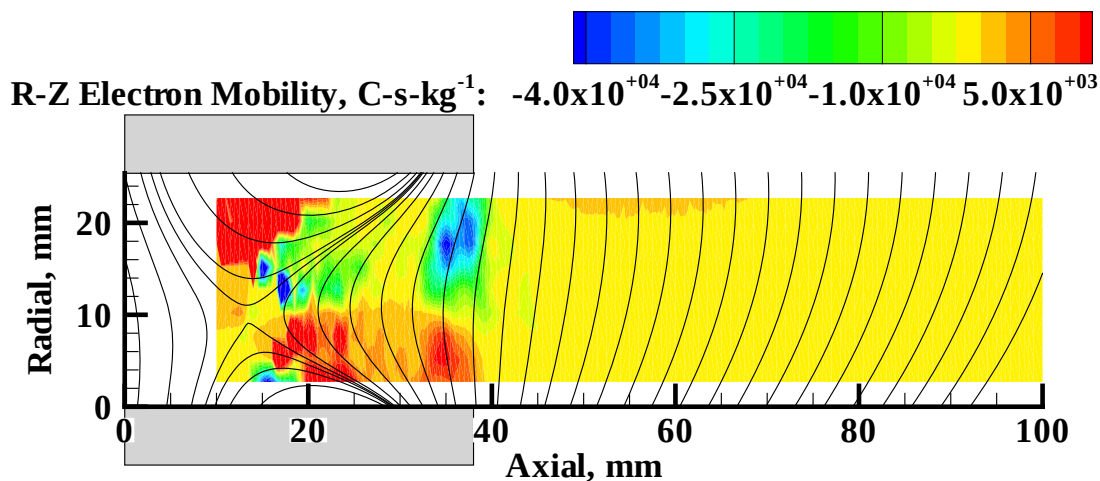


Figure 7. Experimental Electron Mobility for the 500-V Case, a) Axial, b) Radial, and c) R-Z Electron Mobility

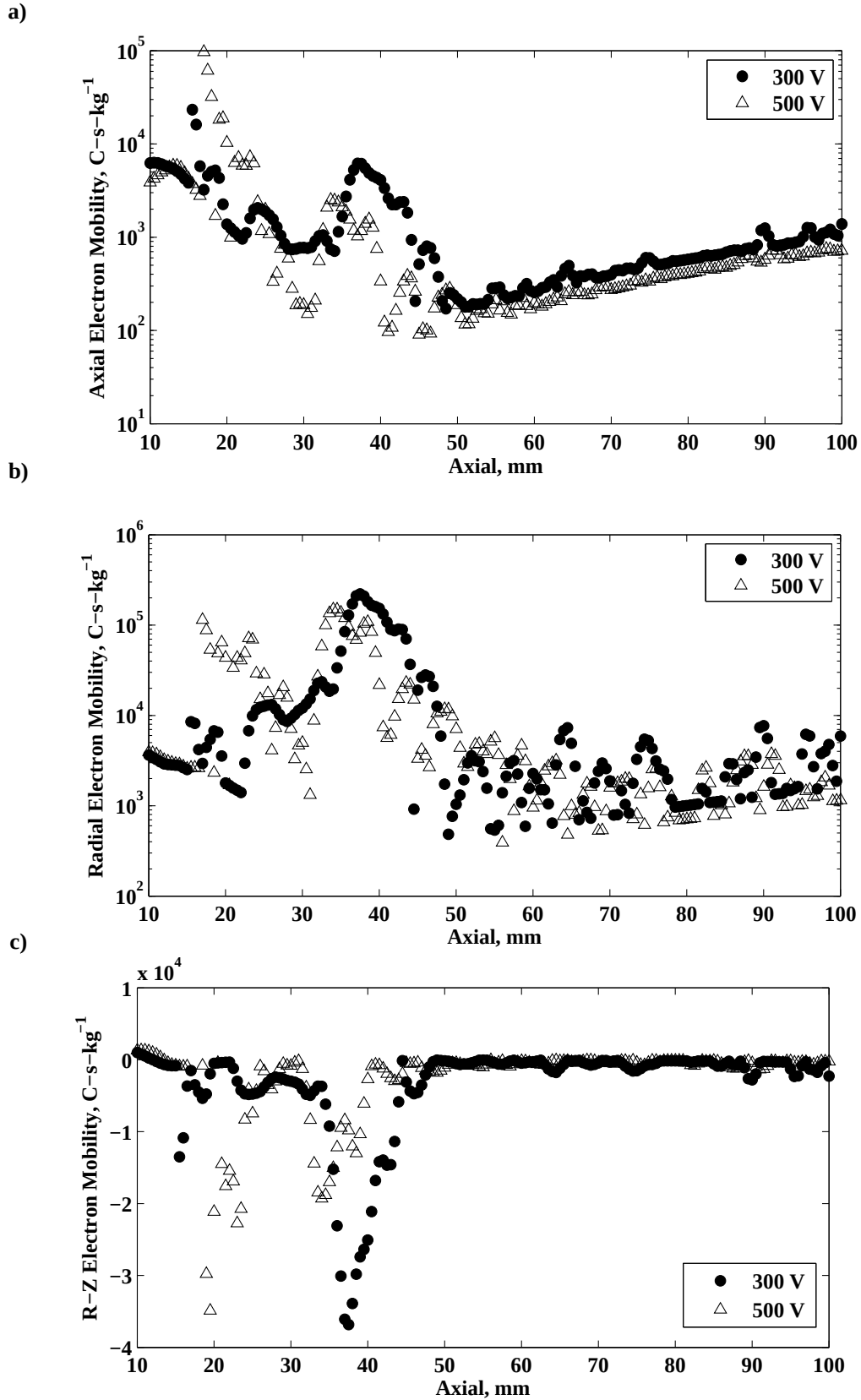
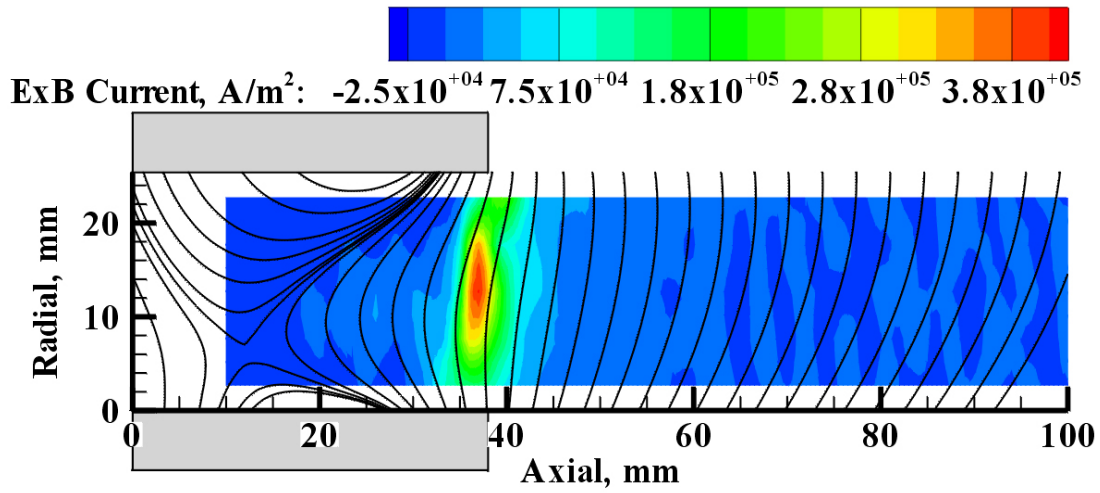
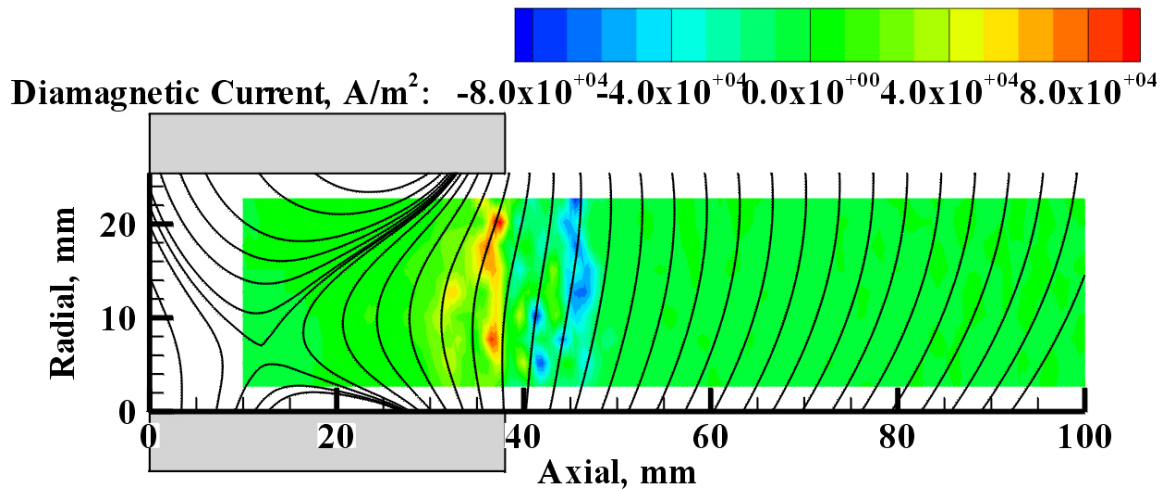


Figure 8. Experimental Results for the 300 and 500-V Cases, a) Axial, b) Radial, and c) R-Z Electron Mobility

a)



b)



c)

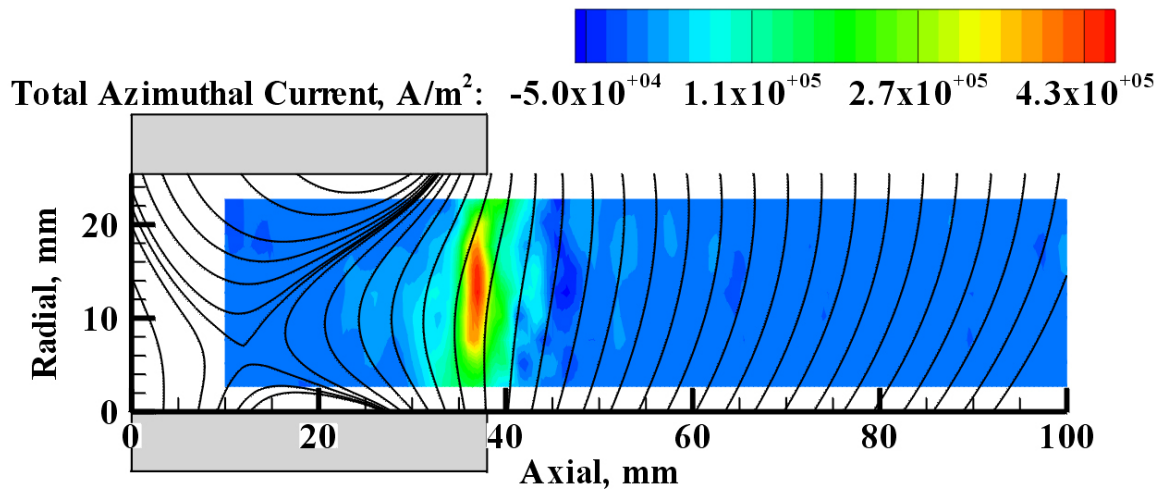
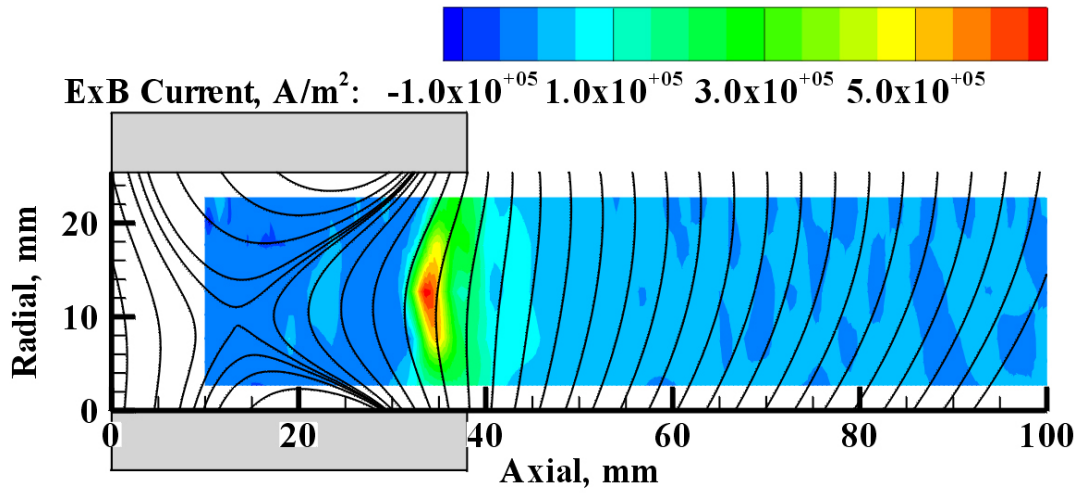
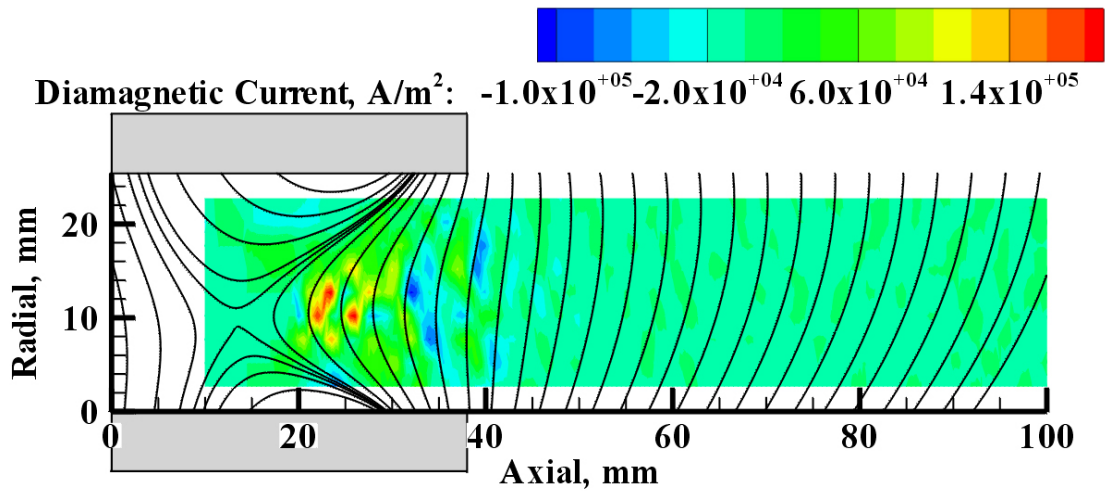


Figure 9. Azimuthal Electron Current Densities for the 300-V Case, a) $E \times B$ Current, b) Diamagnetic Current, c) Total Azimuthal Current

a)



b)



c)

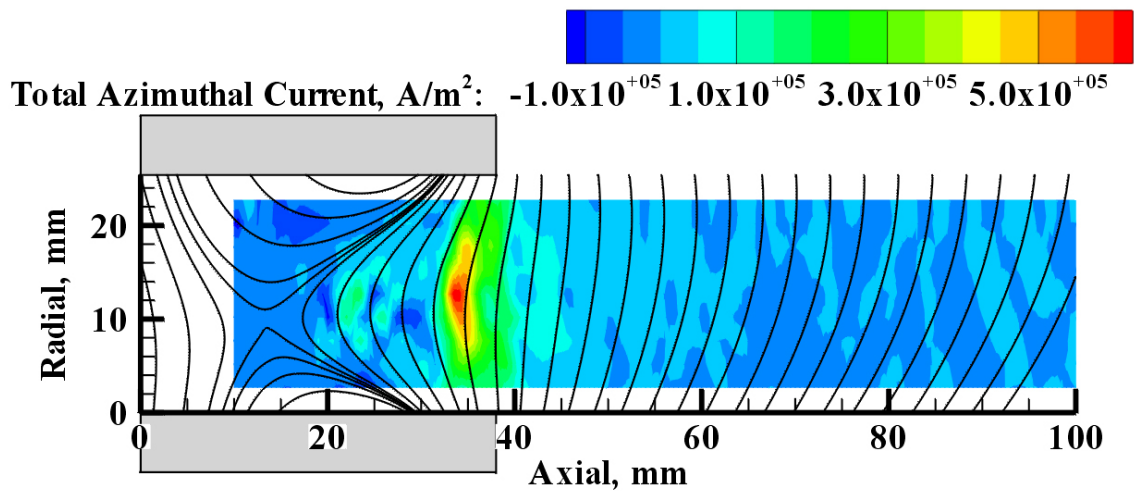


Figure 10. Azimuthal Electron Current Densities for the 500-V Case, a) E×B Current, b) Diamagnetic Current, c) Total Azimuthal Current

V. Conclusion

By incorporating results from internal Hall thruster probe measurements of the NASA-173Mv1 and a fluid description of the electron motion, several electron dynamic properties have been successfully characterized. This analysis is able to calculate two-dimensional mappings of properties including Hall parameter, the total electron collision rate, the electron mobility, and the azimuthal electron currents. While the error in the downstream region is expected to be large, the internal thruster region should be more accurate. It is hoped that this type of analysis can act as a starting point for future Hall thruster experimentation and analysis. Furthermore, the results of this analysis will serve as an aid for Hall thruster modelers during their model development. These experimental results can help to define boundaries and magnitudes for mixed mobility models and serve as a point for comparison in the model development stage.

The electron Hall parameter is found to peak around a value of 400 near the exit of the thruster. The peak Hall parameter region corresponds to a total electron collision frequency minimum of 10^6 Hz. Bohm-like mobility poorly matches these experimental results at the thruster exit although performs better in the downstream region. Electron mobility is strongly affected by the magnetic field strength and field line orientation. The radial electron mobility peaks at the thruster exit and the axial electron mobility peaks near the anode where the magnetic field lines are axial. The total azimuthal electron current density is studied as a combination of $E \times B$ and diamagnetic drifts. The diamagnetic current is found to be approximately 20-25% of the $E \times B$ current.

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