Parametric Investigation of the Rotating Spoke Instability in Hall Thrusters

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This work presents recent results of ongoing investigations into rotating spoke instabilities in Hall thrusters. Recent work by the authors demonstrated large amounts of electron current carried by spokes in the near-anode region in the H6 6-kW laboratory model Hall thruster, and raises the question of whether spokes may play a larger role in electron transport throughout the Hall thruster plume. Spokes have since been observed via high speed imaging in several Hall thrusters, including the H6, the NASA 173Mv1, the X2 dual nested channel Hall thruster, the Helicon Hall thruster, and the Busek BHT-600. While this work focuses on the H6 Hall thruster, visual examples of spoke presence in these other thrusters are also presented. A parameter study of the dependence of spoke amplitude and propagation velocity on magnetic field strength and discharge voltage in the H6 shows that the magnetic field strength has a strong influence on spoke properties. Spokes appear to be omnipresent in Hall thrusters, though with widely variable strength and stability, and appear even at very highly efficient operating conditions with high voltage and high magnetic field strength – the most prominent spoke mode in the H6 is observed at a 600 V, 65% total efficiency operating condition. In response to earlier studies that have observed spokes exclusively at low voltage or in inefficient operating regimes, a study of spoke behavior at a very low voltage operating condition links a transition to higher efficiency operation with the formation of stabilized spoke structures. Finally, we present visual evidence from high-speed imaging of spokes bridging the centrally mounted cathode to the discharge channel.

I. Introduction

In this paper we gather several recent investigations into the rotating spoke instability in the Hall thruster discharge, demonstrating its omnipresence across several Hall thrusters and some of its key characteristics. The rotating spoke instability is interesting because it has been shown to carry a significant portion of cross-field electron current in the near-anode region of the Hall thruster plume[1] and may be a turbulent transport mechanism in the near field plume as well. In this introduction, we describe the motivation for an examination of rotating spokes and review the literature surrounding them. In Section II, we note the experimental equipment used in this work. In Section III, we present results from several investigations into spoke behavior in the H6 6-kW Hall thruster, and finally in Section IV we discuss possible implications of these results and offer two hypotheses regarding the spoke / breathing mode interaction and spoke formation mechanism for future investigation.

A. Prior observations of rotating spokes in Hall Thrusters

The first investigation of a rotating spoke instability in a Hall thruster was by Janes in 1966.[2] Janes' experiment was on an early laboratory model thruster with a tungsten filament cathode, quartz discharge channel walls and a discharge channel length over ten centimeters. In one of the first demonstrations of the inadequacy of classical electron transport mechanisms to explain observed cross-field current in Hall thrusters, Janes used in-situ Langmuir probes and

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floating emissive probes in the discharge channel to demonstrate electron density and plasma potential fluctuations slowly rotating in the $\boldsymbol{E} \times \boldsymbol{B}$ direction with sufficient amplitude to account for the experimentally observed anomalous transport in the thruster. Rotating spokes have since been discussed sporadically in the literature, but only recently have any investigations been attempted that approach the remarkable thoroughness of this early work. With the cessation of Hall thruster funding in the United States by NASA in 1970, there would be no further domestic studies of this mechanism for more than three decades.

In the early 1970s in the Soviet Hall thruster development program Esipchuk further investigated the rotating spoke mechanism in some detail, linking the spoke mode to incomplete ionization and describing its abatement or disappearance at higher power levels.[3] Rotating spokes were found to appear more commonly at low discharge voltages relative to the so-called saturation voltage, which Zhurin describes as the "knee" in a plot of discharge current vs. increasing discharge voltage at fixed magnetic field settings.[4] Further work in this vein likely exists but so far has been difficult to locate in English language translations.

In the United Kingdom in the late 1970s Lomas linked electron current in a high-current-density hydrogen Hall accelerator intended for controlled fusion work to a rotating spoke or "streamer".[5] Lomas detected the spoke via optical and electrostatic probes, measured an azimuthal electric field fluctuation in phase with a density fluctuation in the spoke, and calculated that 20-70% of the 100 A discharge current was carried by this mechanism. Lomas also extended the electrothermal theory of Nelson[6], which had already been applied to streamer modes in MHD generators, to describe a dispersion relation for two modes, a low-frequency spoke mode and a high-frequency (5 MHz) streamer mode. This bifurcation of the azimuthal instability into low- and high-frequency branches is interesting in light of the present study of rotating spokes at kilohertz frequencies and the recent line of work begun by Adam et. al. dealing with azimuthal waves at megahertz frequencies in kinetic models of Hall thrusters and experimental observation of the same instabilities using collective light scattering techniques.[7,8] However, detailed investigation of the possible implications of the model of Lomas and a connection between azimuthal instabilities in different frequency bands observed in modern Hall thrusters lies outside the scope of the present investigation.

Examination of the spoke mode in the United States resumed in the early 2000s at Stanford University, where Chesta observed several spoke-type instabilities in low-voltage Hall thruster discharges of approximately 80-200 V using azimuthally spaced probes, though he did not conclusively link them to electron transport by measuring the relative phase of density and potential oscillations in the spokes.[9] Chesta's efforts are noteworthy as the first since the work of Lomas to attempt a model of the Hall thruster discharge channel in a 2-D axial-azimuthal (z-θ) formulation, using numerical techniques to solve the complicated dispersion relations and carry out stability analyses on the resulting modes.[10] Like the earlier work of Janes and Lomas, Chesta ultimately attributed the spoke formation to electrothermal processes such as ionization, though the detailed mechanisms of this formation were left and still remain unclear. Later work at Stanford by Meezan would experimentally characterize an anomalously high electron mobility near the thruster acceleration region and, by association with large plasma fluctuations also measured in that region, make a correlative argument that the plasma fluctuations were linked to the transport.[11] However, this argument extended well beyond the frequency band of rotating spokes, encompassing all density fluctuations measured by Chesta via a Langmuir probe in ion saturation sampled at 800 kHz.

At Princeton University in 2010 Parker detected a spoke instability on high-speed camera in a small low-power cylindrical Hall thruster (CHT). Spokes were found to be only occasionally present in the CHT, and their appearance linked to a drastic decrease in thruster efficiency associated with large increases in backstreaming electron current.[12] Further work at Princeton by Ellison used in-situ electrostatic probes embedded in the CHT channel to measure density and field fluctuations consistent with axial transport, created a small 4-element segmented anode and estimated approximately half the discharge current passed through a rotating spoke.[13]

In a parallel and independent result, McDonald designed and tested a 12-element segmented anode for the H6 Hall thruster to measure local electron current deposition at the anode, where the entire discharge current is made up of electron current.[1] Discharge current measurements were acquired synchronously with high speed video, matching the current oscillation frequencies with visible spoke rotation frequencies. This experiment concluded that approximately 50% of the electron current in the near-anode region is carried by rotating spokes, a result consistent with the work of Ellison and Lomas. This agreement is remarkable in light of the wildly different thruster sizes, magnetic field configurations, working fluids, and operating regimes of these three thrusters, as well as the different calculation methods used in each case. An additional finding of the H6 segmented anode experiment was that the dominance of the Hall thruster breathing mode oscillation in discharge current measurements made from contiguous anode thrusters is an artifact of the conventional use of a contiguous ring-shaped anode. Measurements on each anode segment showed that the spoke frequency oscillations are the dominant discharge current oscillation at any point around the discharge channel, but since these oscillations are out of phase between different azimuthal locations, they destructively interfere in the summed discharge current around the entire azimuth of the discharge channel. Meanwhile, the breathing mode, while weak at any individual segment of the anode, is in phase around the full 2π of the discharge channel, and sums constructively to appear as the dominant oscillation in conventional discharge current measurements.

B. Prior examination of critical ionization velocity phenomena

Based on their low velocity, Janes and Lowder characterized rotating spokes as part of a larger category of what may be termed “critical ionization velocity phenomena”. The critical ionization velocity was first proposed by Alfvén in 1954, when he speculated that in a collision between neutral gas and a plasma, the plasma should be free to accelerate through the neutral gas until the relative velocity between the species reaches the so-called critical ionization velocity where the kinetic energy of the mobile species reaches the ionization potential of the gas,

$$\frac{1}{2}mv_c^2 = eU_i$$

where m is the mass of the gas, U_i is the ionization potential, and v_c is the critical ionization velocity (CIV).[14] The CIV varies mainly as the inverse root of the atomic mass, since the ionization potential is relatively constant across most gases. For xenon, the CIV is about 4 km/s, while for hydrogen it is about 55 km/s. Once the relative velocity reaches the CIV, any additional energy applied to further accelerate the gas instead drives increased ionization, and the velocity will not increase further until total ionization is reached.

Investigations of CIV phenomena have taken place with some regularity over the years since Alfvén’s first proposal on the subject, and reviews of experimental, numerical and analytical examinations may be found from the early 1970s by Danielsson and Sherman, in 1992 by Brenning, and in 2001 by Lai, among others.[15–18] Experimentally, the CIV is found to hold over a large range of background pressures, discharge currents, and gases. Many of these experiments are performed in homopolar devices, a crossed-field annular geometry similar to a Hall thruster but with an axial magnetic field and radial electric field, resulting in an azimuthal $E \times B$ drift just as in the Hall thruster.

Alfvén notes that a “theory of this [CIV] phenomenon cannot be based on binary collisions, as an atom colliding with an ion has little chance of being ionized unless the velocity surpasses v_c by about a factor of 10. The kinetic energy of the atoms must be transferred to the electrons of the plasma, but a direct transfer by elastic collisions is a much too slow process.”[19] In a recorded discussion of several prominent plasma physicists included with that paper, it is noted that in experiments in the homopolar device the radial current (recall that the homopolar electric field is radial) is observed in discrete surges, suggesting local clouds or blobs of plasma rotating around the azimuth similar to the rotating spoke in Hall thrusters.

Modified two-stream instabilities, lower hybrid instabilities, and more recently general plasma turbulence have been proposed as mechanisms to drive the electron heating that maintains the CIV phenomenon. As Lai notes, “*How to get sufficient turbulence initially needs to be justified on a case-by-case basis*.” In the case of Hall thrusters, experimental measurements of the localized discharge current in the H6 with a segmented anode have demonstrated coherent plasma turbulence associated with rotating spokes, and based on these measurements azimuthal electric fields of the order of the applied axial electric field are suggested,[1] though these azimuthal fields have not yet been measured directly. Such fields would be more than sufficient to accelerate electrons to ionizing energies.

Ultimately, the CIV phenomenon is a general plasma behavior that does not necessarily lend greater insight into the mechanisms of formation or potential means for manipulation of the rotating spoke instability in Hall thrusters. Nevertheless, the wide literature available on this phenomenon may prove fruitful for Hall thruster theorists looking for inspiration, especially considering the small size of the electric propulsion community and its associated body of work.

II. Experimental Setup

A. Vacuum Chamber

All experiments take place in the Large Vacuum Test Facility (LVTF) at the University of Michigan Plasmadynamics and Electric Propulsion Laboratory (PEPL). The LVTF is a 9 m long, 6 m diameter stainless-steel clad vacuum chamber maintained at high vacuum by seven CVI TM-1200 cryopumps with a combined 210,000 L/s pumping speed on xenon. For the 10 mg/s anode mass flow rates with 7% cathode flow fraction used in the majority of the experiments presented in this paper, the background pressure is about 5.9×10^{-6} Torr, corrected for xenon. In the case of the low-voltage transition with elevated cathode flow in Section III.C, the background pressure was slightly higher at 7.3×10^{-6} Torr.

B. H6 Hall Thruster

The H6 Hall thruster is a nominal 6-kW laboratory model thruster with a design operating point at 300 V and 20 A on xenon. The H6 is a joint development effort of the University of Michigan, Jet Propulsion Laboratory and the Air Force Research Laboratory, and a separate copy of the thruster is maintained at each institution. It is notable for its high total efficiency, 64-65% at nominal operation and 70% at 800 V, 6-kW operation.[20,21] For more details on the extensive experimental studies on the H6, we note several recent doctoral theses with more references.[20,22–25]

C. High-speed camera

Without a high-speed camera, spoke detection must rely exclusively on in-situ probes, which provide a window into only a small part of the thruster plasma and make it very difficult to link local oscillations to global phenomena. In-situ plasma probes are also experimentally time-consuming and, in the case of probes mounted internally in the Hall thruster channel, prone to destruction by the plasma. Spoke modes are rarely detectable on discharge current measurements either, since the discharge current is traditionally measured on a ring-shaped anode incapable of resolving current rotation. These hurdles to detection are likely the main reasons why spokes are not often detected in Hall thrusters.

A high-speed camera is a rapid, nonintrusive diagnostic, and it makes spoke detection simpler and less ambiguous than interpreting fixed individual probes or arrays of probes in the plume. With the advent of high-speed imaging technology at plasma timescales (framerates well in excess of the typical 10-30 kHz Hall thruster breathing mode), it becomes possible to examine oscillations across the entire discharge channel literally at a glance.

Two high speed cameras are used in this work. The first is a Photron 1024PCI FASTCAM, capable of 10-bit resolution images at framerates up to 109,000 frames per second (fps) at 128 x 8 pixel resolution, and 27,000 fps at 128 x 128 pixel resolution. The second camera is a Photron SA5 FASTCAM, capable of approximately an order of magnitude higher framerate at comparable image resolution. The SA5 takes 12-bit images at framerates up to 1,000,000 fps at 64 x 16 pixel resolution, and 87,500 fps at 256 x 256 pixel resolution. Both cameras are used with a Nikon ED AF Nikkor 80-200mm lens operated at its maximum aperture of f/2.8. The 1024PCI FASTCAM was used for the parameter study in Section III.B and the images of the H6 and NASA 173Mv1 in Figure 2, while the SA5 FASTCAM was used for all other images.

The FASTCAM views the thruster axially through a quartz viewport with an interior sacrificial glass plate cover from approximately 6.5 meters downstream. The thruster is aligned on chamber centerline while the camera viewport is approximately 1 meter off chamber centerline, making for an alignment angle within 10 degrees of axial. In the horizontal plane the viewport is raised above the thruster mounting surface and all high speed video and images are taken from approximately 2.5 degrees above the horizontal level of the thruster.

III. Experimental Results

A. Rotating Spokes as an omnipresent feature of Hall thruster operation

Rotating spokes of varied strength and stability have been observed on every Hall thruster and at every operating condition imaged at PEPL. To date this includes 5 different Hall thrusters: the H6, the NASA 173Mv1,[26] the dual nested channel X2 Hall thruster,[27] the Helicon Hall thruster operating in pure Hall mode,[28] and the BHT-600. We define the spoke mode number m as the number of simultaneous spoke excitations present at one time. Since spoke modes will be discussed at length in the following sections, Figure 1 illustrates the physical meaning of the spoke mode numbers.

Postprocessed still frames of the thrusters imaged at PEPL are shown in Figure 2, generated according to the process described by McDonald.[1] More images of the H6 similar to those in Figure 2 have also been published.[1,29] Images of the BHT-600 were taken only in a small slice across the discharge channel, making the spoke difficult to see in still images (though it is clear in video), and so are not shown here. The channels of the X2 are postprocessed into false color individually, since the channels have distinct breathing mode frequencies and thus require distinct

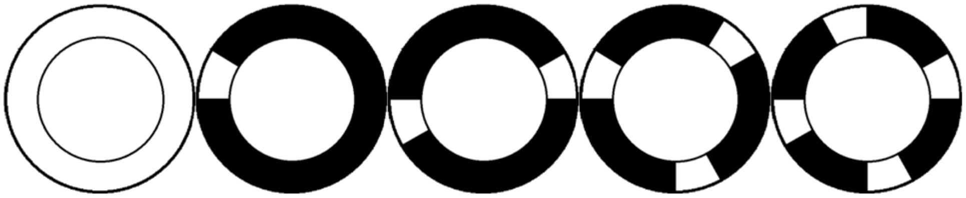


Figure 1. Physical meanings of the various spoke modes. White indicates luminous spokes propagating through an otherwise dark background. The mode number m indicates the number of spokes simultaneously present. For $m = 0$, the entire channel flashes in unison. For $m \geq 1$, the spokes propagate azimuthally.

normalizing functions to visualize their respective spokes.

In addition to these thrusters, similar high-speed imaging work at Princeton by Parker and Ellison has detected a spoke mode in a small cylindrical Hall thruster,[12,30] and Liu at the Air Force Institute of Technology has recorded azimuthal nonuniformities propagating at kilohertz frequencies in the Busek BHT-200.[31].

Different spoke modes may appear in the same thruster at different operating conditions, and likewise in different thrusters different spoke modes may appear even at the same operating condition. In fact, spokes are in many cases not perfectly stable and adjacent spoke modes may "trade off" unpredictably, such that an $m = 3$ mode may transition into an $m = 2$ mode and back into an $m = 3$ mode, all over a span of milliseconds. The implications of this in the Fourier spectra of the high-speed video are discussed in Section 1.

Higher spoke modes appear to form in physically larger thrusters; for example, in smaller thrusters such as the BHT-600 or in Princeton's CHT only an $m = 1$ mode has been observed, while in the X2 outer channel, the largest thruster yet imaged, the highest mode numbers of $m = 5 - 6$ appear. In the middle of the size range, the H6, 173Mv1 and HHT demonstrate modes between $m = 2$ and $m = 4$. The variation of spoke mode number with operating condition in the H6 is discussed in Section B.

In the literature of rotating instabilities the sign of m often denotes the handedness of rotation, for example positive m as rotation in the right-handed direction about a magnetic field. Since spoke rotation changes direction with the polarity of the B-field, we denote positive spoke modes $m \geq 1$ as rotating in the $E \times B$ direction. While we have not observed $m < 0$ (e.g., a spoke mode rotating in the anti-$E \times B$ direction) in the H6 or any of the other Hall thrusters mentioned above, Smith has reported the detection of such a mode in the Stanford Hall thruster using floating emissive probes to measure plasma potential fluctuations.[32]

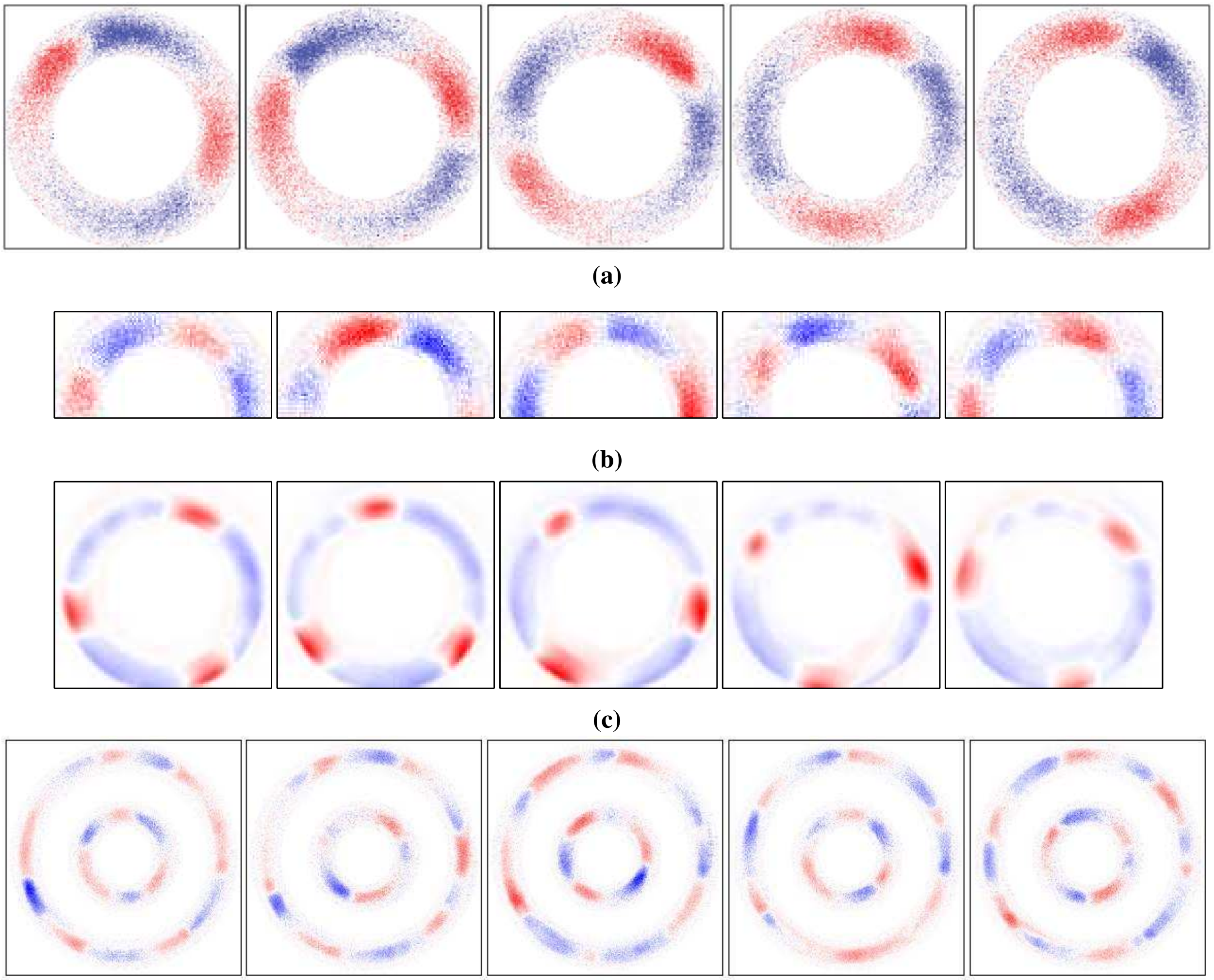
B. Parameter Study

Given the omnipresence of rotating spoke modes across several different thrusters and operating conditions, we conducted a parameter study to examine the spoke mode number, amplitude and propagation velocity's dependence on magnetic field strength and discharge voltage. Our examination consists of three datasets: a sweep of magnetic field strength at constant discharge voltage, a sweep of discharge voltage at constant magnetic field strength, and a sweep through discharge voltage with magnetic field settings optimized to minimize discharge current at each voltage. These three datasets are shown in the parameter space of B and V_D in Figure 3.

At each operating condition between one half and one full second of video was acquired at 27,000 fps with 128x128 pixel resolution on the 1024PCI FASTCAM. These videos were in turn used to generate 2D DFTs of the high-speed video, with the results presented in Figure 5. Synchronous discharge current measurements were also collected for these conditions, but are not presented since in general the discharge current signal DFT is very similar to the $m = 0$ video signal DFT. The results are summarized in terms of the magnitude and frequency of the peaks for each mode.

The amplitude of the mode is given in arbitrary units, since it is a measurement of the pixel intensity signal recorded on the high-speed camera CCD. This signal is a function not only of thruster operating condition, but also camera aperture and framerate, and so in general is not directly comparable across different videos. The camera framerate and aperture are held constant at 27,000 fps and f/2.8 to maintain comparable signal amplitudes within this study.

It is important to realize also that the raw pixel output of the camera is not a direct measure of visible light intensity. Non-linearity in the bit depth of the camera sensor causes the pixel output to asymptote and saturate at high incident fluxes. This can be accounted for by calibration if necessary. However, for the particular cases of imaging the Hall



(a) **(b)** **(c)** **(d)**

Figure 2. Postprocessed video frames showing rotating spokes in several Hall thrusters: (a) the H6 operating at 300 V and 10 A with a $m = 2$ spoke mode, imaged at 27000 fps; (b) the NASA 173Mv1 operating at 300 V and 7 A with a $m = 3$ spoke mode, imaged at 45000 fps (only part of the thruster was imaged to achieve a higher framerate with the 1024PCI camera); (c) the Helicon Hall thruster (HHT) operating at 200 V and 23.5 A with a very clearly delineated $m = 3$ spoke mode, imaged at 162,500 fps with only every fourth image shown; (d) the X2 dual nested channel Hall thruster operating at 150V, 6.6 A on the inner channel and 16.8 A on the outer channel, with an $m = 3$ mode on the inner channel and a $m = 5 - 6$ mode on the outer channel. The spokes counter-rotate in the X2 due to reversed magnetic field direction between the channels.

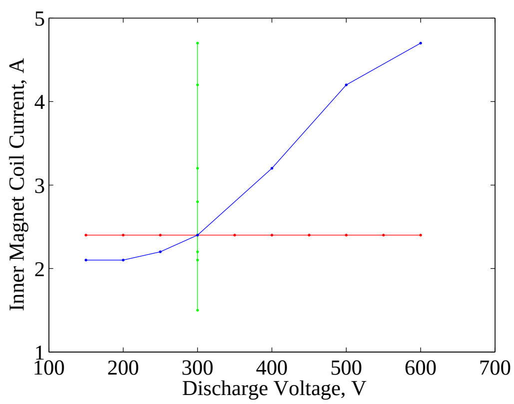


Figure 3. Parameter space for H6 operating regime investigation of rotating spokes.

thruster presented here, the high frame rates used are sufficient to keep the light intensity reaching the camera well inside the linear regime of the CCD. The exception to this rule is the cathode, which is often an order of magnitude brighter than the discharge channel and is generally saturated in high speed video. Since the cathode portion of the frame is not used for analysis, it does not affect the results presented here.

The linear velocity v_m of spoke passage around the discharge channel for a spoke mode m (that is to say, a spoke mode with m spokes simultaneously present) is given for all $m \geq 1$ by its frequency peak f_m as

$$v_m = \frac{2\pi R f_m}{m}$$

For the H6 a $m = 1$ spoke mode with $f_1 = 1$ kHz travels at an approximate velocity v_1= 500 m/s. The $^1/_m$ term in the linear velocity equation is because the 2D DFT gives a local frequency at a fixed azimuthal location – if m spokes are present, the local frequency is boosted by a factor of m compared to the global rotation frequency of any single spoke. Thus, a $m = 2$ spoke mode with $f_2 = 1$ kHz in the DFT only has a linear spoke velocity of 250 m/s, compared to 500 m/s for the $m = 1$ mode with the same frequency. Put another way, the $m = 2$ mode with $f_2 = 1$ kHz is actually composed of two spokes, each propagating at 500 Hz but spaced 180 degrees around the discharge channel so together they appear as a 1 kHz propagating wave at any fixed observation point in the channel. An isolated probe in the channel cannot distinguish between modes with different global frequencies that manifest at the same local frequency, but high speed video of the full discharge channel confidently distinguishes between different spatial modes in the same frequency range.

1. Interpretation of the 2D DFT

To further clarify the interpretation of the 2D DFT, we examine two demonstrative cases in Figure 4. The left DFT of Figure 4 is taken for a 600V, 10 mg/s operating condition. This is one of several regular operating points in the H6 throttling table and is described in more detail by Reid.[20] This condition has a 65% total efficiency, yet harbors the strongest, most coherent spoke mode yet observed. The $m = 3$ and $m = 2$ spoke modes are strongest, at 8 and 3 kHz, respectively, and the $m = 3$ mode peak is over an order of magnitude higher than the $m = 0$ breathing mode, typically thought of as the strongest oscillation in the Hall thruster discharge. The appearance of several smaller peaks below the $m = 3$ peak is an artifact of turbulent unsteadiness in the mode's propagation. Even a stable mode may occasionally lose a spoke for a few microseconds, or gain an extra spoke in the spoke structure, but the missing spoke may reappear and/or the extra spoke die out sufficiently quickly that the velocity of the underlying stable mode is unaffected. Thus, strong peaks often "smear" into adjacent mode numbers, though the dominant peak with the highest amplitude is generally easy to spot. This plays out in the DFT on the left as spurious $m = 2$ and $m = 4$ modes with the same frequency as the $m = 3$ mode.

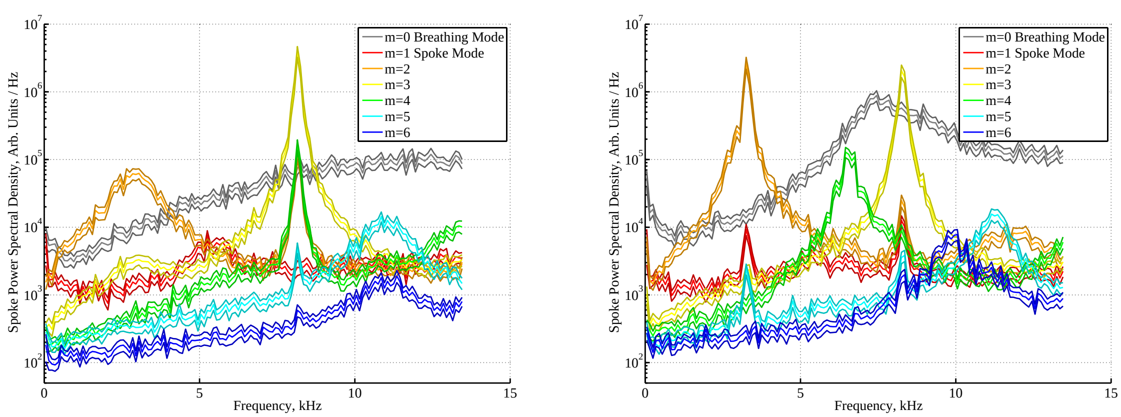


Figure 4. Example 2D DFTs illustrating several subtle features of mode analysis. The left DFT, taken from a 600V, 10 mg/s operating condition with nominal magnetic field, shows a dominant $m = 3$ mode with turbulent smearing into the adjacent $m = 2$ and $m = 4$ modes. The right DFT, taken from a 300V, 10 mg/s operating condition with slightly elevated magnetic field, shows some turbulent smearing but also beat modes and spatial harmonics of the fundamental modes.

The second DFT at right in Figure 4 is of a 300 V operating condition at 10 mg/s with a 3.2 A inner magnet current, slightly above the typical magnetic field strength for this point in the H6 throttling table. This mode illustrates several interesting features that complicate analysis of a 2D DFT. The $m = 2$ and $m = 3$ modes are still strongest, though the breathing mode is also relatively strong at this operating condition. Turbulent smearing of the $m = 2$ mode and the $m = 3$ modes is present, but several new features have crept in as well. The $m = 4$ and $m = 6$ peaks near 6.6 and 9.9 kHz are in fact the second and third spatial harmonics of the $m = 2$ mode at 3.3 kHz, while the $m = 5$ mode at 11 kHz is a beat frequency mode formed by the $m = 2$ and $m = 3$ modes – the mode number is the sum of the parent modes, while the linear velocity calculated from the frequency is the average of the parent mode linear velocities. Harmonic, beat and turbulent smearing modes are not included in the parameter study results table, since they are artifacts of the fundamental modes introduced through the Fourier decomposition.

2. Parameter Study Results

The results of the parameter study are presented in Figure 5. The results are presented in three columns corresponding to the three datasets mentioned previously. We note before presenting the results that this assessment is intended to be more qualitative than quantitative; in particular, for many points the interval between data collection at different conditions was sufficiently brief due to test window constraints that an assumption of steady-state conditions was not rigorously verified. The low Nyquist frequency of the camera framerate (13.5 kHz for 27 kfps) also makes it difficult to identify higher spoke modes than $m = 3$ in many cases. Nevertheless, from the large selection of data we call attention to several key features:

1. **Higher spoke modes travel faster.** The clearest trend in the results is that the $m = 3$ modes always travel faster than $m = 2$ at the same operating condition, and likewise where observed the $m = 4$ mode travels faster than $m = 3$. The $m = 4$ mode is only observed in the handful of cases where it drops below the Nyquist frequency of 13.5 kHz, for example in at the highest magnetic field settings in the first column of Figure 5.

2. **Spokes in general appear more strongly and stably at higher magnetic field strengths.** A sharp, narrow peak in the DFT like the $m = 3$ mode in the top of Figure 4 indicates a very stable, coherent mode. These sharp peaks tend also to be the largest in magnitude, while the more broadband peaks (as for example the $m = 2$ mode in the same figure) tend to be of lower magnitude. Above about 2.5 A of magnetic coil current, or above 300V in the optimized-field parameter sweep, the spoke modes achieve parity or surpass the magnitude of the $m = 0$ breathing mode peak. This trend is not observed when the discharge voltage alone is increased, even though this might have been expected since at higher discharge voltages the electron temperatures in the plume should also be raised, increasing the likelihood of visible emission from excited ions and neutrals. The effect appears to be a function primarily of magnetic field. Since the magnetic field suppresses axial electron mobility, this may indicate that spokes strengthen in response to large gradients in plasma potential and plasma and neutral densities, or possibly that with the increased Hall parameter at higher fields that the spokes become a larger force in maintaining the discharge.

3. **Higher spoke modes become more dominant at higher magnetic field settings.** This holds true in both the magnetic field sweep at constant voltage and in the optimized operating conditions, where we note from Figure 3 that the B-field strength increases approximately linearly with voltage. This is especially noticeable in the two highest magnetic field settings and two highest voltages with optimized magnetic field settings, where the $m = 3$ mode overtakes the $m = 2$ mode in amplitude by one to several orders of magnitude. The $m = 3$ rotational frequency falls below the breathing mode frequency at these points, perhaps suggesting that a spoke frequency below the breathing mode frequency enables strong spoke modes. The $m = 3$ spoke velocity also falls below the breathing mode at several operating conditions in the 100-300 V range as well, but at these lower voltages the spoke modes are so unstable and thus weak in the DFT that it is hard to draw many conclusions. We will address low voltage operation further in the next section.

4. **Spoke velocity appears to decrease slightly and/or asymptote at higher magnetic fields.** This trend is clearest in the magnetic field sweep at constant voltage, but it also holds in the optimized operating conditions. The velocity is in general between several hundred and a few thousand meters per second, or between about 10-40% of the critical ionization velocity in xenon, a result consistent with previous results dating back to the initial findings by Janes and Lowder, who recorded values about 20% of the CIV. For comparison, the $E \times B$ velocity in the discharge channel is on the order of 10 km/s, so the spoke mode is much slower than the $E \times B$ drift.

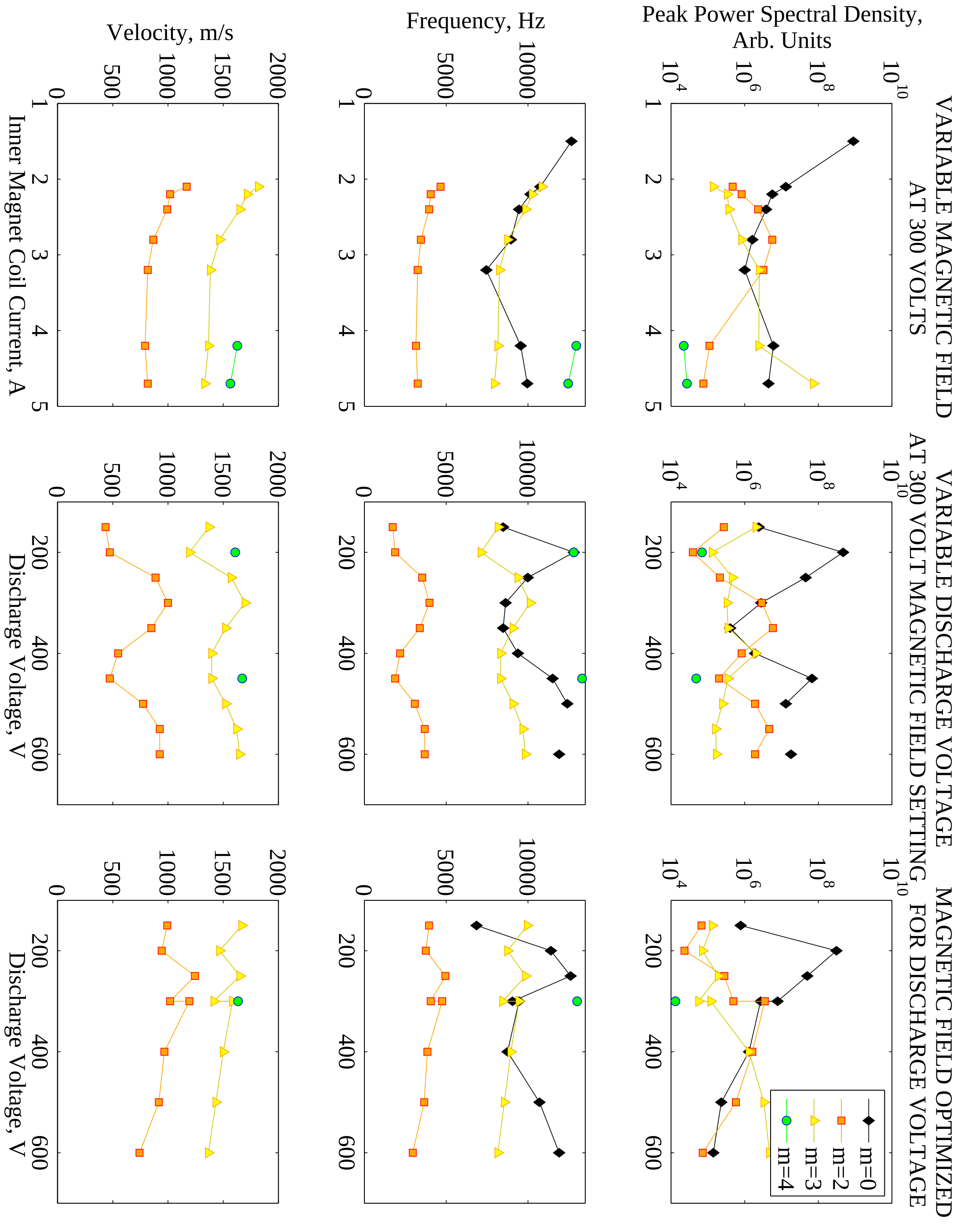


Figure 5. Parameter Study Results

C. Rotating spokes in low voltage operation

Early results in the rotating spokes literature generally found the spokes to occur at low voltage operating conditions. However, the parameter study of Figure 5 demonstrates that spoke presence and behavior in the H6 tends to be more a function of magnetic field strength than discharge voltage, and in fact for optimized magnetic field settings even high-voltage operating regimes can have strong and stable rotating spokes. The parameter study also demonstrates that at low voltages and low magnetic fields, below say 300V or 2.4 A, the breathing mode tends to be large relative to the spoke modes, even for optimized magnetic field settings. In this section we examine very low voltage operation at about 100 V, below the range explored in the parameter study and pushing the limits of stable operation of the H6.

Sharp transitions in a low-voltage operating mode in the H6 have been described by Brown,[23] and have since been observed on both other copies of the H6 at the University of Michigan and Jet Propulsion Laboratory. This transition was well-characterized by Brown in the range from 100-120V, where depending on cathode flow rate or magnetic field the discharge current of the thruster can be drastically affected in step-function fashion. These sudden shifts in discharge current are comprised largely of changes in backstreaming electron current through the plume, raising efficiency by several percent while maintaining constant thrust. Hysteresis in the thruster discharge makes it possible to operate in either mode at certain choices of cathode flow fraction or magnetic field, depending on the direction of approach. Based on consistent discharge current oscillation frequencies between modes, though with very widely different amplitudes, Brown attributes these shifts to one of several causes: rotating spoke instabilities, cathode oscillations, power supply oscillations, or the Hall thruster breathing mode. Using the high-speed camera we are able to link these shifts with the formation of a coherent rotating spoke instability.

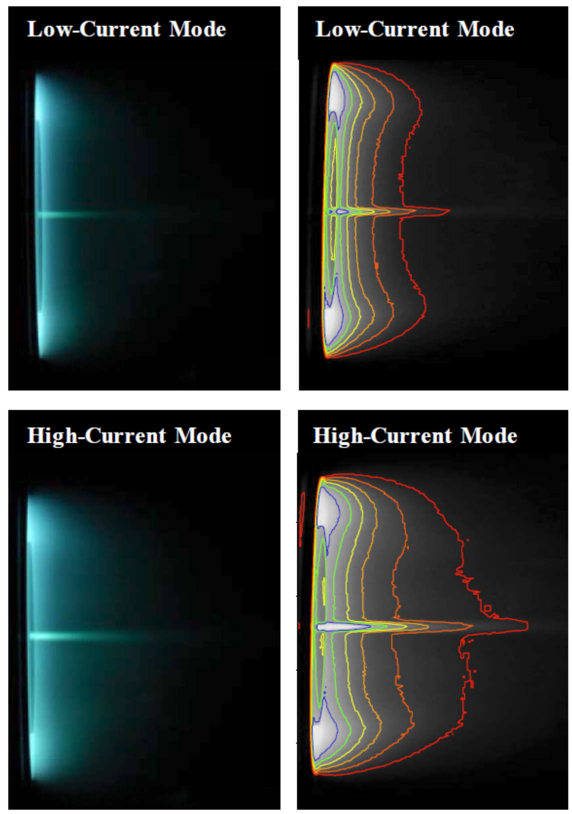


Figure 6. Figure and caption reprinted from Brown[23]: Photographs of the [H6] 6-kW Hall thruster jet-mode plume structure for the low-current mode (top left) and high-current mode (bottom left) during 105-V, 20-mg/s operation. Contours of constant image intensity are shown for the low-current mode (top right) and the high-current mode (bottom right). Photos were taken with identical settings on a tripod mounted Nikon D200 DSLR using a 70- mm lens with manual focus at F/5, ISO-400, and exposure time of 1/2000 sec.

Experiments on the University of Michigan copy of the H6 reproduced a low-voltage transition similar to the one observed by Brown, albeit at a higher cathode flow fraction (CFF), about 35% CFF. This high cathode flow fraction is due to conscious choice not to use a supplemental trim coil to optimize magnetic field for low voltage as Brown did. This maintains a consistent magnetic field topology with all other operating conditions examined in this work. While this cathode flow fraction is very high, it should also be noted that Brown demonstrated the transition at 14% CFF but observed a relatively flat thruster efficiency of 33% from 14% CFF up to 26% CFF. Performance measurements were not taken during high-speed imaging, but the discharge current behavior observed clearly indicates Brown's transition.

The transition is between two modes labeled "high-current" and "low-current." The high-current mode is characterized by extremely wide oscillations in discharge current, with mean discharge current 10.6 A and peak-to-peak current oscillations on the order of 20 A. The DFT is shown at top left in Figure 7, where the sharp breathing mode peak at about 5 kHz and several higher harmonics are clearly visible. The low-current mode, triggered at higher cathode flow fractions, has a mean discharge current of only 9.4 A and peak-to-peak current oscillations of about 2 A. At top right in Figure 7, the low-current mode's breathing mode peak is at approximately the same frequency but with a power spectral density many orders of magnitude lower. This quiescent low-current mode also shows a visible change to a more focused plume structure, seen in Figure 6.

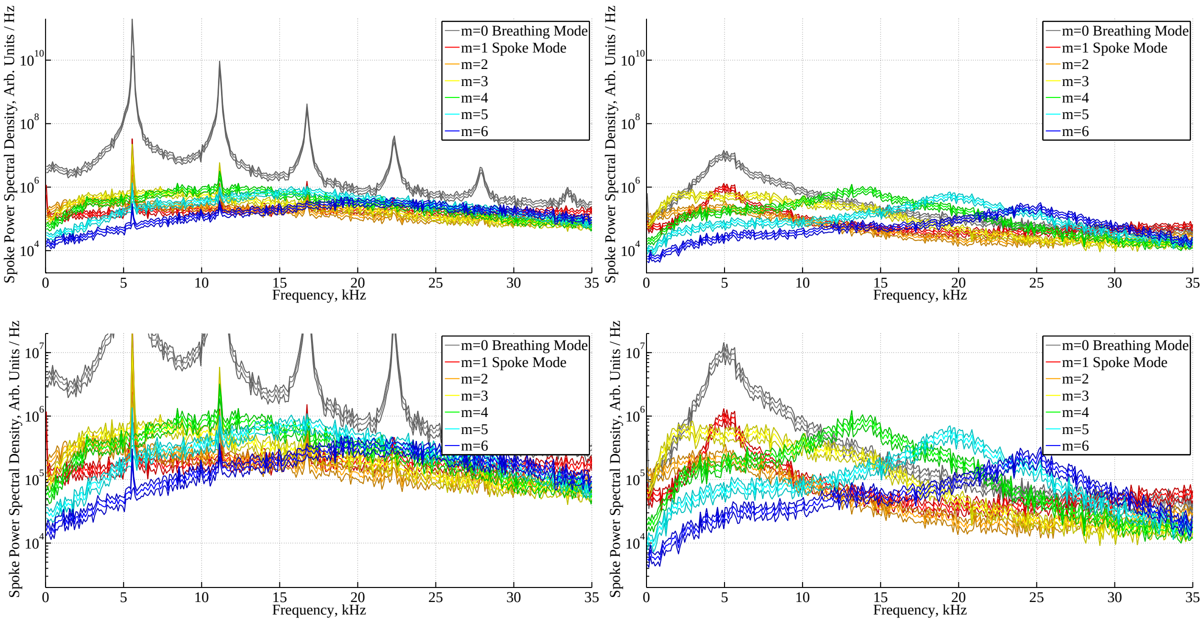


Figure 7. Selected 2D DFT power spectra observed in operating mode transitions at 105V, 10 mg/s. Top left, high-current mode operation with a dominant $m = 0$ breathing mode oscillation; top right, the low-current mode showing a much more quiescent oscillation spectra. Bottom, enlarged images of both spectra show the shift in spoke mode structure from broadband frequency structures in the high-current mode to clearly peaked structures in the low-current mode, indicating the formation of more stable and coherent spokes. This transition from an exaggerated breathing mode with decoherent spokes to a quiescent discharge with more stabilized spokes corresponds to a reduction in backstreaming electron current and an improvement in thruster operating efficiency.

Variation in the spoke modes $m \geq 0$ is less evident than the drastic reduction in the dominant $m = 0$ breathing mode oscillation, but on closer examination these structures also change with the mode transition. The enlarged images at bottom in Figure 7 show that, while the peak power spectral density amplitude for the spoke modes does not appreciably change during the transition, the overall structure of the power spectral density does change. In particular, during the low-current mode the spoke mode frequency peaks are much more clearly defined. In the context of the 2D DFT, this means that in the low-current mode the structures are more coherent, holding together in a wave structure for longer sustained periods. In the high-current operating condition there are four-, five- and six-fold rotating structures in the discharge channel, but the wide frequency hump indicates that they are not coherent – instead, they turbulently form and reform at different points in the discharge channel, giving rise to effective discontinuous pulses of spoke behavior that are represented with this broadband frequency spread.

From this data alone it is not clear whether the change in spoke behavior causes the mode transition or is caused by the mode transition, i.e., whether the relationship is causal or merely correlated, but it does demonstrate that low-voltage operation is more efficient when stable spokes are present. For 105 V operation Brown measured an improvement in total efficiency from 31% to 33% and in thrust to power from 79 to 86 mN/kW between the high-current and low current mode.

D. Spatial extent of spokes - radial bridge from cathode

Given that rotating spokes are omnipresent across different thrusters and operating conditions, can effect strong changes in thruster performance at least at low voltage depending on their stability, and contribute significantly to cross-field electron transport in the near-anode region, it is reasonable to ask if they are a candidate electron transport mechanism outside the discharge channel as well. To this end we review some evidence that the spokes extend spatially into the near-field plume.

High-speed imaging of the discharge channel necessarily axially integrates a signal through the entire plume from the camera's downstream viewpoint, making it difficult to gauge the axial extent of the spokes. However, this vantage point is capable in principle of resolving the radial extent of the spokes. The effort is complicated by very low visible light intensity over the inner and outer pole pieces due to the low plasma densities in this region compared with the discharge channel.

Following the implementation of Fourier analysis in high-speed axial imaging of a Hall thruster in McDonald,[1] we consider a video as a 3D function for pixel intensity vs. pixel row, column, and time. Since we focus on azimuthal spoke behavior, the Cartesian row-column coordinates of the image are converted to polar coordinates centered on the discharge channel and the radial dependence of brightness is neglected to create a 2D function of pixel brightness vs. azimuthal angle and time. We reproduce Eqn. 11 from McDonald here:

$$p_{norm_{AC}}(i, j, k) = p_{norm}(i, j, k) - M(i, j) \tag{1}$$

In this equation $p_{norm_{AC}}$ is the quantity plotted in the still frames of Figure 2. The variable p_{norm} is a normalized three-dimensional pixel brightness function of pixel row i, pixel column j, and video frame k. The "normalization" is to improve spoke visualization by removing the breathing mode oscillation signal. The breathing mode manifests in video as an overall brightening and dimming of the discharge channel, so the normalization brightens "dim" video frames and dims "bright" video frames such that the summed pixel intensity for each video frame is constant in the normalized video. The quantity M is a mean image composed of the average intensities of each pixel over the course of a video.

We define a new quantity, the normalized fluctuation intensity, as

$$\widetilde{p}_{norm_{AC}}(i, j, k) = \frac{p_{norm_{AC}}(i, j, k)}{p_{norm_{AC}Pk-Pk}(i, j)} \tag{2}$$

where the AC-coupled oscillations in each pixel are divided by the peak-to-peak oscillation amplitude on each pixel over the course of the video. Note that we have now applied two distinct normalizations to the initial video signal! The first was applied in time, to brighten dim frames and dim bright frames, and this second normalization from Eqn. 2 is applied in space, to put the dim regions of each image over the pole pieces on an equal footing with the bright regions of each image over the discharge channel.

The result is shown in Figure 8 for H6 operation at 300 V and 10 mg/s with the segmented anode. The upper series of images in the figure shows the quantity $p_{norm_{AC}}$ from Eqn. 1, while the lower image shows $\widetilde{p}_{norm_{AC}}$ from Eqn. 2. Without the re-scaling relative to the local fluctuation amplitude, the regions over the inner pole are barely visible in the upper image. With re-scaling, the lower image shows a clear spoke-like structure emanating from the centrally mounted cathode and bridging across the inner pole to the discharge channel. Since the absolute amplitudes of the oscillation over the pole are small, they appear to discontinuously cut off and shift azimuthally at the discharge channel inner radius. However, this appearance is consistent with a continuous structure bridging the cathode to discharge channel projected onto the plane of the image, where the faint portion of the bridge in the plume over the discharge channel could be washed out by the bright anode end of the spoke structure in the discharge channel.

Not every Hall thruster video shows this clear presence of spoke structures over the inner pole; indeed, this case is one of only a very few times they are so clearly visible. Nevertheless, the fact that such structures can be observed visually *at all*, especially given the low signal to noise ratio in this region, is highly motivating for direct investigation with electrostatic probes as spoke structure may be present in this region even in cases where it is not so clearly visible.

There is already some such evidence for rotating instabilities extending out into the plume in this frequency range. Smith observed rotating structures in the plasma potential at about 25 kHz (using a small thruster where higher frequencies would be expected for km/sec type velocities) extending out into the plume.[32] Time-resolved investigations by Lobbia several thruster diameters downstream of the BHT-600 have also shown unexplained kilohertz-level oscillations in plasma density and potential not associated with the breathing mode that may be evidence of rotating spokes.[33]

IV. Discussion

From the previous section, we have the following pieces of information: rotating spokes are omnipresent in the Hall thruster discharge, they appear even at highly efficient operating conditions, they account for a substantial level of electron current in the near-anode region, and there is very suggestive evidence from both video and in situ probes that they may extend throughout the plume, radially bridging the cathode to exit plane across the near field and extending axially downstream as well.

A theoretical framework for how spokes affect overall thruster operation and in particular how they interact with the axial Hall thruster breathing mode is likely necessary to fit all of these pieces together. This work does not

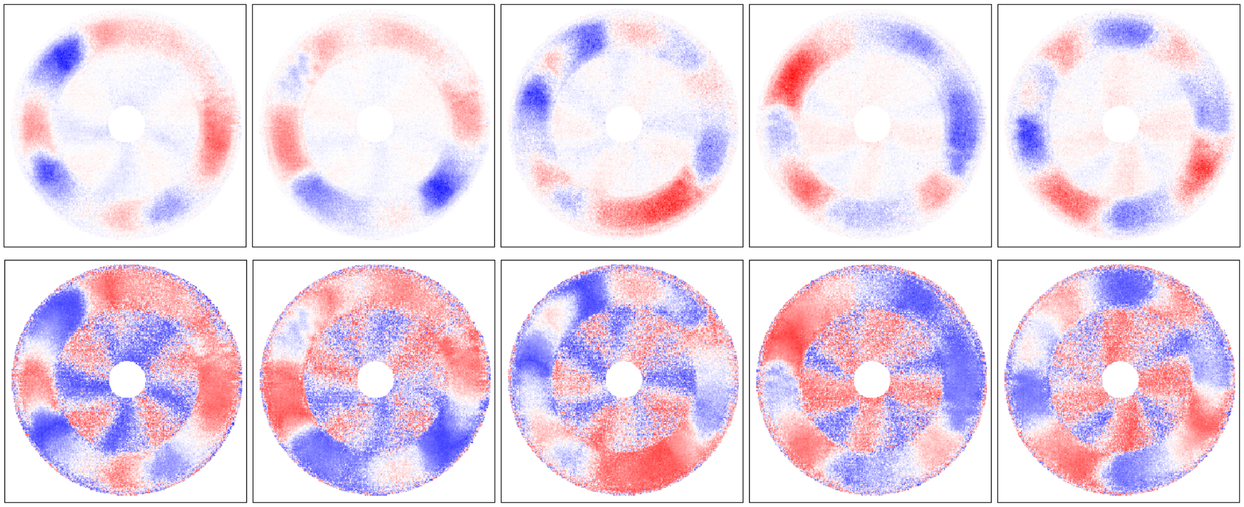


Figure 8. Top, a postprocessed set of still frames show the H6 operating at 300V, 10A with the segmented anode, imaged at 87.5 kfps. Bottom, the same frames are plotted but showing the normalized fluctuation amplitude in Eqn. 2. This rescaling of the visible signal based on the peak-to-peak oscillation amplitude of each pixel makes the radial extent of the spoke structure visible even in the very dim regions over the inner pole, and show that the spokes can form a bridge between the cathode and the exit plane across the near field. Note that the central cathode is still cropped from both image series, since the bright cathode discharge saturates the camera sensor and oscillations are not discernible over the saturated pixel signal.

present an analytical model or self-consistent theory, but we do outline some physical hypotheses based on these empirical observations that we hope may prove illuminating or inspiring for the future development of such a model. Ultimately, these are hypotheses in the full sense of the word: attempts to explain the observed physical phenomena that require further testing to be confirmed or, quite possibly, rejected. As Choueiri notes in his overview of Hall thruster oscillations, "*the detailed physics of this [rotating spoke] mode in the Hall thruster plasma remain largely unexplored*,"[34] and the thoughts below are intended as fruit for discussion and motivation for further exploration.

A. On the relation between thruster size, power level and spoke behavior

A central contradiction a suitable spoke hypothesis should resolve is the difference in early results observed with spokes, where they generally were observed more prominently at low voltage and inefficient operating conditions, compared with the bulk of the present results that document spoke presence and even dominance at high voltage, high efficiency operating conditions.

We suggest that the difference in results may be due to the difference in power level and more fundamentally the difference in physical size between the H6 and the thrusters of previous studies. In particular, we hypothesize that in general the spoke mode is neutral or beneficial to Hall thruster operation when it can propagate at a frequency at or below the Hall thruster breathing mode frequency, while it tends to be detrimental to thruster operation in cases where it can only exist at higher frequencies.

Consider the scaling of three important quantities: the thruster channel radius, which generally scales together with thruster power; the breathing mode frequency, which is relatively constant in the 5-35 kHz range across most thruster power levels and sizes, though it tends to increase with discharge voltage; and the spoke frequency $f = v/2\pi r$, which due to the small variation of spoke velocity across a wide range of operating parameters mainly scales as the inverse of the thruster radius. Since the spoke travels in a narrow velocity range, and since spoke velocity rises with the spoke mode number, it stands to reason that in thrusters with a small radius even the slowest mode $m = 1$ may propagate at such high frequency that it exceeds the breathing mode frequency. The spoke clearly creates an ionization and excitation front as it travels around the channel, hence the visible emission detected on the high speed camera, and if this ionization front travels azimuthally faster than the neutral replenishment rate, there is the potential for one mode to starve the other of neutrals, though how to decide which mode survives and which is starved is not clear.

As an example of one extreme, a dominant breathing mode with practically no spoke mode takes place in the H6 at the 105 V high-current mode operating condition where the Brown transition is observed. The spoke modes that appear most prominently in the DFT all have frequencies greater than the 5 kHz breathing mode frequency, and the breathing mode completely takes over the discharge, oscillating with enormous amplitude. When the higher frequency spokes are stabilized by a large cathode flow fraction, thruster efficiency improves and the breathing mode is tempered significantly, decreasing from a 20 A oscillation to only about 2 A. This high-current condition is the closest to a no-spokes condition observed, and the very large breathing mode is reminiscent of SPT-100 operation. At the other extreme, the highest voltage conditions with optimized magnetic field in the parameter study where the spoke modes grew very strong and existed well under the breathing mode are the highest performance conditions in terms of overall efficiency observed in the H6.

Some support for the general idea of plasma fluctuations damping the breathing mode may be found in recent work in HPHall2De by Mikellides on the H6 Hall thruster.[35] In particular, Figure 18 in that work shows the effect of a reduced Bohm collision frequency in the thruster plume on discharge current oscillations. Reducing the Bohm factor α from 0.15 to 0.075 in the plume while suppressing Bohm diffusion entirely inside the channel in both cases resulted in a large increase in discharge current oscillations, from a peak-to-peak amplitude on the order of 10A on a 15A discharge to about 1 A, similar to the effect on discharge current oscillations in the Brown transition.

B. The cathode region as a potential source of the rotating spoke instability

To date most investigations of the rotating spoke have focused on the near-anode region, especially recently through the use of the segmented anode by McDonald and Ellison.[1,30] However, while the spoke is present and easily detectable in this region, several factors suggest that it may originate or be critically dependent upon regions at the hollow cathode. First, the changes in Hall thruster operation associated with the Brown transition at low voltage are associated with conditions near the cathode induced by increased neutral density flowing either directly through the cathode as an increased cathode flow fraction or indirectly through auxiliary flow injected around the cathode.[23] Second, the images in Figure 8 visibly show spokes emanating from (or, admittedly, possibly extending into) the cathode. This shows that at least at some operating conditions that spokes exist in this region, though it is not conclusive that they are always present or sourced from here.

The third and perhaps most compelling piece of evidence linking spoke formation to the cathode or near-cathode region is an early work in a Hall accelerator by Allario that isolated the formation of a rotating spoke instability to the behavior of magnetic field in a critical region near a tungsten filament cathode.[36] Allario's thruster used a circular tungsten filament extending fully around the annular discharge channel as a cathode. By balancing the applied radial magnetic field with the self-field created by the heating current in the tungsten filament, Allario located a critical region close to the cathode where the cancellation of the two magnetic fields triggered the suppression of the spoke instability, which otherwise formed in the usual $E \times B$ direction with the highest plasma density near the cathode. This work also postulated a link with earlier work by Simon relating the criteria for spoke formation to the applied electric field and axial density gradient.

The geometry of a modern Hall thruster is quite different from the arrangement of Allario, since for a centrally mounted cathode the cathode current is ejected axially along the magnetic field, and the magnetic field bows out and is nowhere purely radial except in a narrow region of the discharge channel near the exit plane. As a result the cathode self-field and the applied field do not cancel, and instead are for the most part mutually orthogonal in the cathode region. Nevertheless, the behavior is similar to observations by Brown that, in addition to cathode flow variation, magnetic field variation can also trigger mode transitions. While transitions triggered by magnetic field variation were not explicitly studied for spoke behavior with the high-speed camera, the work of Brown shows them to be similar in every respect to transitions triggered by cathode flow variation, and it seems plausible to assume that they are also correlated with the forced stabilization of spoke modes.

A possible physical narrative for the formation of the spoke in the cathode region may be found in the instability-rich geometry of an arc along the axis of a magnetic field. In particular, the radial density gradients in neutral and plasma density in the hollow cathode region can induce Rayleigh-Taylor instability, while the radial electric field coupled with the axial electric field and resulting plasma rotation can induce Kelvin-Helmholtz instability as well. Both instabilities may drive the formation of fluting in the cathode discharge column. The azimuthal self-field of the hollow-cathode discharge, which is an appreciable fraction of the applied axial magnetic field near the cathode orifice, adds to the applied field to create a helical total magnetic field. The combination of fluting of the hollow cathode discharge and swirling due to the helical total magnetic field might be sufficient to form spoke precursors emanating from the cathode like the image in Figure 8.

V. Conclusions

The rotating spoke instability is an omnipresent feature in Hall thruster operation, observed across a range of Hall thrusters and a variety of operating conditions. The strength and stability of the mode varies across a continuum, from cases where spoke modes are coherent and clearly visible as in Figure 2, sometimes visually of larger amplitude than even the thruster breathing mode, to cases where spokes are decoherent, short-lived, not apparent in still video frames and really only detectable in the 2D DFT of high-speed video. Whether this second regime should truly be categorized as a spoke instability or merely labeled as low-frequency, long-wavelength turbulence is a matter for debate and difficult to answer confidently without a clear understanding of what factors stabilize turbulence into coherent, periodic modes.

Previously the rotating spoke was generally associated with inefficient thruster operation, but the most coherent, largest amplitude spoke mode yet observed in the H6 Hall thruster has been at the highest efficiency operating condition imaged to date, a 600 V 6-kW condition with a total efficiency of 65%. At low voltage the stabilization of the rotating spoke instability is associated with a transition in Hall thruster operating mode associated with a decrease in electron current and corresponding improvement in thruster efficiency. This result is in contrast to previous results by Parker in a cylindrical Hall thruster where the formation of spoke modes was detrimental to Hall thruster performance and associated with increased electron current.[12]

While recently documented experiments have focused on spoke presence in the near-anode region of the discharge channel, visual evidence from high-speed imaging of the H6 as well as plume measurements by Smith suggest a wide extent of the spoke in the radial and axial directions from the discharge channel.[32] In particular, Figure 8 shows the spokes bridging the centrally mounted cathode of the H6 to the discharge channel across the near field.

The underlying physics of the spoke mode in the Hall thruster and its influence on Hall thruster performance remain uncertain. Two hypotheses are presented for discussion and as suggestions for further testing: first, the potential link between physical thruster size and the interaction between the spoke mode and breathing mode, motivated by the different results observed previously with small low-power thrusters and those reported here in the large high-power H6, and second, the possibility of spoke formation in the near-cathode region, motivated by low-voltage operating mode transitions correlated with changes in spoke stability and similar results previously reported by Brown and Allario.[23,36]

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