Preliminary Magnetohydrodynamic Simulations of Magnetic Nozzles

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We present preliminary results from two computational magnetohydrodynamic (MHD) studies of magnetic nozzles. The first study models the magnetic nozzle expansion of a plasma into a near vacuum with a resistive MHD solver. Results from resistive MHD simulations suggest that further model development is necessary to replicate experimental results within the desired physical regime. The second study investigates the flow of a magnetically guided plasma into a background plasma using a generalized Ohm's law MHD solver. Results from the generalized Ohm's law MHD simulations of the non-expanding jet are obtained using the Magneto-Gas Kinetic Method. Generalized Ohm's law simulations show bulk rotation of the plasma jet and the formation of helical structures in the velocity and current due to the Hall effect.

Nomenclature

ρ, n	Mass Density, Number Density
T	Temperature
v, U	Velocity
e	Energy
B	Magnetic field
ω	Frequency
q	Charge of Electron
g_0	Acceleration Due to Gravity at Surface of Earth
L	Characteristic Length
p	Pressure
m	Mass
J	Current Density
E	Electric Field
τ_{col}	Collision Time
ν	Collision Frequency

Subscript

$\perp$	Perpendicular to Magnetic Field
e, i	Electron, Ion
p	Plasma
$\parallel$	Parallel to Magnetic Field
c	Cyclotron
f	Flow Characteristic

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I. Introduction

Strong guiding magnetic fields known as magnetic nozzles have been suggested for the purpose of plasma flow control and thrust generation in electric propulsion devices. This technology is incorporated in numerous advanced propulsion systems such as magnetoplasmadynamic thrusters (MPD's),[1–4] helicon and radio frequency thrusters,[5–7] and the VAriable Specific Impulse Magnetoplasma Rocket (VASIMR)[8,9] which enable future space missions

Devices that utilize magnetic nozzles are a part of a class of electric propulsion devices known as electromagnetic propulsion devices. Thrust is generated in electromagnetic propulsion devices through interactions between plasma currents and magnetic fields that may be externally applied or internally induced.[10] This is distinctly different from electrostatic propulsion devices which generate thrust by the interaction between an electric field created by the device and charged particles.

The approximate operating regimes of existing and proposed propulsion devices are shown in Figure 1 in terms of thrust and specific impulse (v_{exit}/g_0). The electromagnetic regime in this figure reflects a desired operating regime which also extends into the electrostatic regime. An ideal propulsion method maximizes both specific impulse and thrust while real devices are limited by power and material constraints. Electromagnetic propulsion methods that utilize magnetic nozzles are attractive for future space missions due to the predicted high specific impulse operation, limited interaction of hot plasma with electrodes, and promising scaling to higher thrust densities than existing electric propulsion devices.

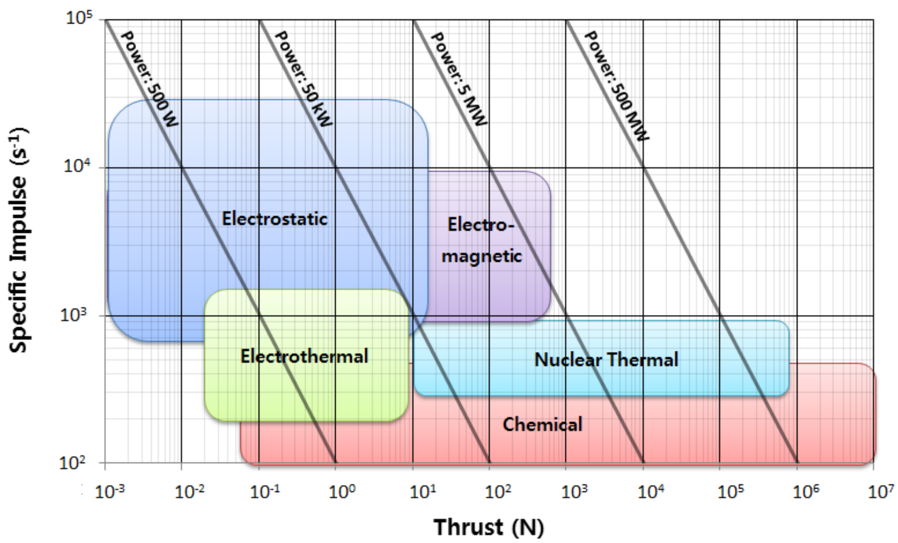

Figure 1. Specific impulse and thrust operating regimes for space propulsion methods with power level contours.

This paper presents results from preliminary computational studies of magnetic nozzles using a magnetohydrodynamic description of the plasmadynamics. Sections III further introduces magnetic nozzle physics, magnetic nozzle propulsion devices, and the computational methods which have been previously used to study magnetic nozzle physics. Section IV presents the preliminary results from magnetohydrodynamic simulations. Section V concludes the paper.

II. Overview of Magnetic Nozzles

This section introduces magnetic nozzles and propulsion devices which utilize them. The numerical methods that have been used to study magnetic nozzle physics are also discussed.

A. Introduction to Magnetic Nozzle Physics

Magnetic nozzles are functionally similar to de Laval nozzles by achieving thrust generation through conversion of internal energy or non-directional kinetic energy of the plasma to directed kinetic energy. The virtue of magnetic nozzles lies in minimizing contact between the high temperature plasma and surfaces while also providing mechanisms for thrust generation by plasma-field interaction. Magnetic field topology and thereby the magnetic nozzle configuration is also variable, enabling versatility in nozzle shape. A comparison between de Laval nozzles and magnetic nozzles is shown in Figure 2.

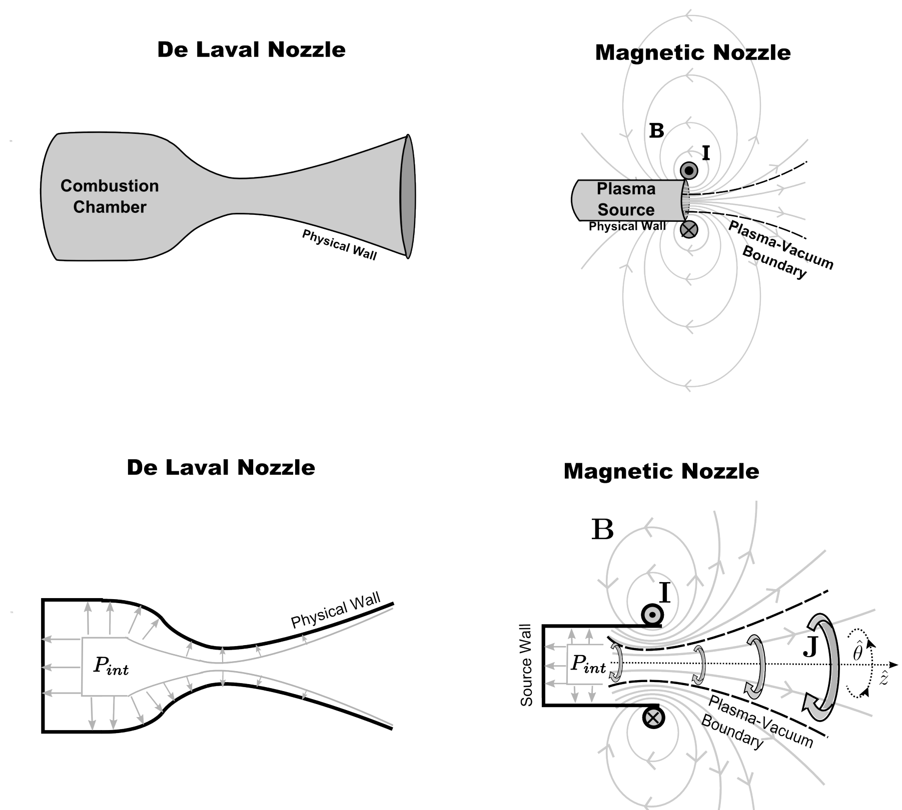

Figure 2. De Laval nozzle compared to magnetic nozzle produced by current loop with current I.

Three crucial steps are required to generate thrust in a magnetic nozzle:

1. Conversion of internal energy and non-directed kinetic energy to directed kinetic energy
2. Momentum transfer from the plasma plume to the spacecraft
3. Efficient detachment from the applied field with minimal plume divergence

Energy conversion mechanisms in magnetic nozzles include: A) conservation of magnetic moment adiabatic invariant; B) Hall acceleration; C) Thermoelectric acceleration; D) directionalizing fluid thermal energy; E) Joule heating.[11–16] Momentum transfer results from a combination of the pressure forces on the plasma source and the interaction between the magnetic field and the induced currents in the plasma plume shown in Figure 2.[14,17–22] The currents induced in an expanding magnetic nozzle plasma jet are primarily azimuthal due a combination of the azimuthal symmetry of the nozzle and the tendency of the applied magnetic field to exert a force on the plasma as it flows or expands across the magnetic field. Detachment methods can be grouped into three categories: collisionless, collisional, and magnetic reconnection detachment. The primary means for achieving collisionless detachment are due to particle drifts, loss of adiabaticity, electron inertial effects, and induced magnetic field effects.[23,24] Collisional detachment may be achieved through resistive diffusion across magnetic field lines,[25–27] recombination and charge exchange collisions,[28,29] and current closure.

Magnetic fields must initially confine the plasma plume to a configuration which produces directed kinetic energy. The plasma must then efficiently detach from the closed applied magnetic field lines which may pull the plasma back to the spacecraft. This duality of requiring confinement and separation presents the primary challenge in magnetic nozzle design. The transition from plasma containment to detachment must be understood to optimize magnetic nozzles.

B. Propulsion Devices Utilizing Magnetic Nozzles

The concept of the magnetic nozzle was first introduced by Andersen[30] and has since been incorporated in a number of potential plasma propulsion devices. Helicon and radio-frequency sources are currently being developed into thrusters such as the Helicon Double Layer Thruster (HDLT) by the incorporation of a magnetic nozzle downstream of the plasma source.[5] These thrusters show promise for future small satellite (CubeSats, microsattellite) missions in power ranges near 10-100 W. Research into helicon thrusters has significantly improved the understanding of thrust generation in magnetic nozzles.[5,6,22,31] Higher power helicon sources have also been suggested for use in magnetic nozzle propulsion devices.[7] The VASIMR propulsion system generates plasma by a high power helicon source.[8,9] The plasma is then further heated by an ion cyclotron resonance heating stage before undergoing the magnetic nozzle expansion. The VASIMR experiment currently operates at 200 kW producing approximately 6 N of thrust.[32] Scaling to higher power levels up to 1 MW is feasible, making VASIMR a candidate for a number of missions ranging from station keeping, lunar tug, and even manned missions. Implementation of a magnetic nozzles has also been considered for MPD thrusters to limit interaction of the plasma with the walls to increase lifetime of the thruster and improve thrust generation. Future fusion based propulsion systems also utilize magnetic nozzles for thrust generation.[15,16] In these theoretical devices the energetic plasma generated by the fusion process is expanded by a magnetic nozzle. It is predicted that these devices would operate at 1 GW generating 4.6 kN of thrust.[33] Fusion devices are not included in Figure 1 and would significantly extend the regime of electromagnetic propulsion. It has to be noted that the development of all of these thrusters is intimately coupled to the development of the necessary power sources.

C. Numerical and Theoretical Study of Magnetic Nozzle Physics

The numerical and theoretical methods used to study magnetic nozzle physics span a broad range of models which include single-fluid magnetohydrodynamics (MHD), two fluid plasmadynamics, kinetic theory, and particle descriptions.

Single-fluid MHD models have been used in a number of studies.[26,34–36] Steady-state, ideal MHD methods were used to study the super-Alfvenic detachment of plasma from magnetic nozzles.[34] A resistive, time-accurate MHD solver known as the Multiblock Arbitrary Coordinate Hydromagnetic (MACH) code has been used in a number of studies of magnetic nozzle physics .[35,37] Previous parametric studies have suggested that a generalized Ohm's law MHD model should be used to capture additional important physical mechanisms of the thrust generation process in many current magnetic nozzle experiments such as the HDLT and VASIMR.[38,39] A generalized Ohm's law method includes two fluid effects through incorporation of the Hall term and electron pressure effects.

The plasmadynamics in magnetic nozzles has also been described by models which treat the electrons and ions as separate fluids.[7,17,23] Studies with these models generally use steady-state approximations and include assumptions that make the solutions computationally tractable. These studies have significantly improved the understanding of the full thrust generation process.

Particle descriptions have also been utilized to study important physical phenomena in magnetic nozzles.[40–43] Particle-in-cell (PIC) methods which treat the electrons and ions as particles have been used to study the formation of double layers in magnetic nozzle plasmas.[42,43] PIC simulations have also been used to study the detachment process.[40,41] The use of these methods could be further expanded to study the full thrust generation process.

III. Preliminary Magnetohydrodynamic Simulation Results

We perform preliminary simulations of magnetic nozzle plasma jets with resistive and generalized Ohm's law MHD solvers. This study is performed to study the thrust generation process and to examine two fluid, Hall effects in magnetic nozzles. The results from these studies are summarized in this section.

A. Governing Equations

The magnetohydrodynamic (MHD) equations describe the dynamics of plasma flow as a single fluid. The principal assumptions of magnetohydrodynamics are that the electron behavior is fast compared with the bulk fluid motion and that the electrons inertia can be ignored. This leads to the simplification of the more complex multi-fluid description to a single fluid description through the simplification of the electron momentum equation into an Ohm's law. The MHD conservation equations are closed by the Ohm's law which relates the electric field, $\boldsymbol{E}$, the magnetic field, $\boldsymbol{B}$, and the current density, $\boldsymbol{J}$. The generalized Ohm's law is shown in Equation 1.

$$\boldsymbol{E} = -\boldsymbol{U} \times \boldsymbol{B} + \frac{1}{n_e q}\boldsymbol{J} \times \boldsymbol{B} - \frac{1}{n_e e}\nabla(n_e k T_e) + \eta \boldsymbol{J} \quad (1)$$

In this equation $\boldsymbol{U}$ is the center of mass velocity of the ions and electrons, n is the number density, T is the temperature, q is a unit of charge, and k is the Boltzmann constant. The terms on the right side of Equation 1 will be referred to as the convective, Hall, electron pressure, and resistive terms respectively. The full generalized Ohm's law must be used to capture many of the important physics of magnetic nozzles.[39] For the results presented here we utilize both a generalized Ohm's law solver which includes all terms and a resistive MHD solver which includes only the convective and resistive terms.

The mass, momentum, and energy conservation equations for single fluid MHD are shown in Equations 2 - 4. The momentum and energy equation are written in the magnetic pressure formulation with total pressures, $p_{tot} = p + B^2/2\mu_0$, and total energies, $e_{tot} = \rho U^2/2 + p/(\gamma - 1) + B^2/2\mu_0$, which include the contribution of the magnetic field. In these equations $\underline{\underline{\tau}}$ is the dissipative stress tensor.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \quad (2)$$

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot [\rho \mathbf{U}\mathbf{U} + -\mathbf{B}\mathbf{B}/\mu_0] = -\nabla(p_{tot}) + \nabla \cdot \underline{\underline{\tau}} \quad (3)$$

$$\frac{\partial e_{tot}}{\partial t} + \nabla \cdot \left[\mathbf{U}(e + p_{tot}) - \frac{\mathbf{U}\cdot\mathbf{B}}{\mu_0}\mathbf{B}\right] = \nabla \cdot (k \nabla T) + \nabla \cdot (\mathbf{U} \cdot \underline{\underline{\tau}}) - \nabla \cdot \left(\frac{\mathbf{J} \times \mathbf{B}}{\mu_0 \sigma}\right) \quad (4)$$

The magnetic induction equation and Ampere's law are shown in Equations 5 and 6 respectively. These equations close the system of equations leading to a self-consistent description of the plasma flow and magnetic field evolution.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad (5)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad (6)$$

In depth discussion of the generalized Ohm's law and the resistive MHD solvers used for these numerical simulations can be found in previous papers.[38, 44–47]

B. Resistive Magnetohydrodynamic Simulations

The computational method used for the resistive MHD simulations is a 2 $^1/_2$-D axisymmetric solver. This solver is extensively validated and has been used to study high density thermal plasmas used in plasma accleration devices.[44–46] In these simulations the plasma jet expands out of an inlet into a low background pressure gas. A magnetic field generated by a current loop is applied over the domain and is allowed to develop in time. The boundary around the inlet is a dielectric wall while the far field utilizes extrapolation boundaries. The electrical conductivity is a constant classical Spitzer conductivity calculated based on the inlet parameters. The parameters chosen are near the operational regime of the VASIMR experiment and are shown in Tables 1 and 2.

The results from cases with no magnetic field (NMF), low initial magnetic field (LMF, $B_0 = 0.002$ (T)), and high initial magnetic field (HMF, $B_0 = 0.02$ (T)) will be discussed. Simulations are run to the same total time at which the flow is nearing steady state. The domain is mirrored across the axis of symmetry in the figures below to better visualize the results. Figure 3 shows contours of density with velocity stream

	Gas	n $(\#/m^3)$	T_i (eV)	T_e (eV)	U (m/s)	L_{char} (m)
VASIMR[48]	Argon	$\approx 10^{17} - 10^{19}$	$\approx 10^1 - 10^2$	$\approx 10^0 - 10^1$	$\approx 10^4$	$\approx 10^1$
Numerical Exp.	Argon	$1 \cdot 10^{19}$	$2.4 \cdot 10^1$	$2.4 \cdot 10^1$	$3.2 \cdot 10^4$	10^1

Table 1. Magnetic nozzle experiment comparison

Mach	Magnetic Reynolds	Pressure Beta	Kinetic Beta	Ion Hall Par.	Ion Residence Par.
$M = \frac{U}{\sqrt{\gamma RT}}$	$UL\sigma\mu_0$	$\beta_p = \frac{nk_BT}{B^2/2\mu_0}$	$\beta_f = \frac{\rho u^2/2}{(B^2/2\mu_0)}$	$\Omega_{col,i} = \frac{\omega_{ci}}{\nu_i}$	$\Omega_{f,i} = \frac{\omega_{ci}L}{U}$
3.25	$4 \cdot 10^3 - 4 \cdot 10^4$	$2.4 \cdot 10^{-1} - 2.4 \cdot 10^1$	$2.3 \cdot 10^0 - 2.3 \cdot 10^2$	$2.4 \cdot 10^0 - 2.4 \cdot 10^1$	$6 \cdot 10^{-1} - 6 \cdot 10^0$

Table 2. Magnetic nozzle experiment comparison

lines. These results show radial stretching of the density profile and an expanded velocity streamline fan due to the stronger magnetic field. Computational problems are evident near the axis where the density profile does not round out as expected.

Figure 4 shows axial velocity contours with velocity streamlines. The jet widens with increasing magnetic field strength, but shows no significant decrease or increase in axial velocity due to the application of the magnetic field. This suggests that the nozzle is not completing its primary function of further accelerating the flow. The stream lines also suggest that the radial velocity is increased as the magnetic field strength is increased.

Axial magnetic field contours (B_z)with magnetic field stream lines are shown in Figures 5. NMF cases are not shown since no magnetic field is applied. The left set of two figures correspond to LMF cases and the right set of two figures correspond to the HMF cases. In each set, the left figure corresponds to the initial time step and the right plot corresponds to the final time step. The presence of the flowing plasma significantly decreases the axial magnetic field and alters the magnetic field stream lines. This behavior results from the high conductivity of the plasma and is characterized by the high magnetic Reynolds number.

The flow of the plasma generates currents which are shown in Figure 6. Paramagnetic currents (positive $\hat{\theta}$) are shown in the top half and diamagnetic (negative $\hat{\theta}$) currents are shown in the bottom half. Paramagnetic currents are generated primarily at the edge of the plume while diamagnetic currents are generated near the axis. The $\mathbf{J} \times \mathbf{B}$ force due to the interaction of the paramagnetic currents and the magnetic field results in the expansion of the jet profile. The paramagnetic and diamagnetic currents account for both the reduction of the magnetic field as well as the turning of the magnetic field lines.

These results represent a low order approximation and do not incorporate all of the necessary physics to study magnetic nozzle physics within the regime modeled.[39] The numerical method generally performs well, but the physical model chosen is not appropriate for the chosen regime. These results do not compare well with the most recent experimental results and should not be a reflection of the magnetic nozzle performance. A number of computational issues were encountered when trying to simulate within this regime which will be addressed in future simulations. These results are more thoroughly presented in a previous paper.[49]

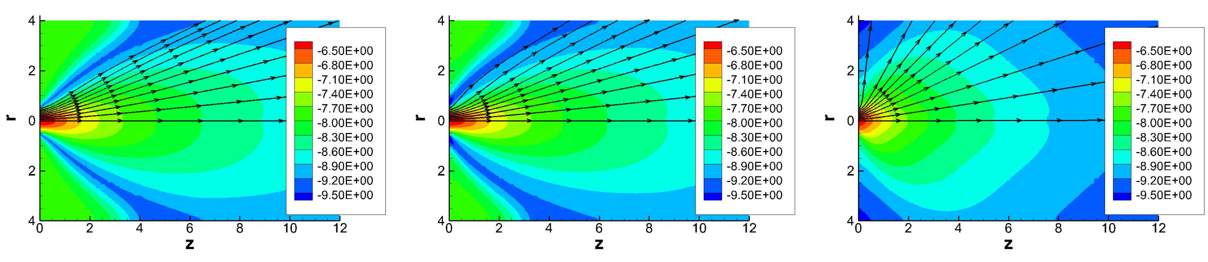

Figure 3. Density contours ($log_{10}(kg/m^3)$) with velocity stream lines. Left: No magnetic field, Center: $B_0 = 0.002$ (T)**, Right:** $B_0 = 0.02$ (T)

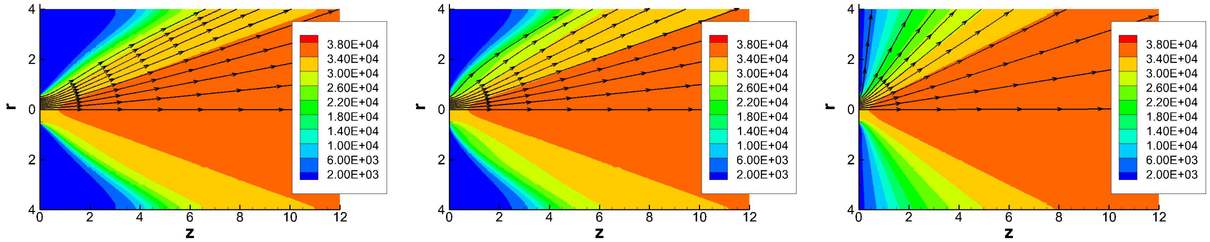

Figure 4. Axial velocity contours (m/s) with velocity streamlines. Left: No magnetic field, Center: $B_0 = 0.002$ (T), **Right:** $B_0 = 0.02$ (T)

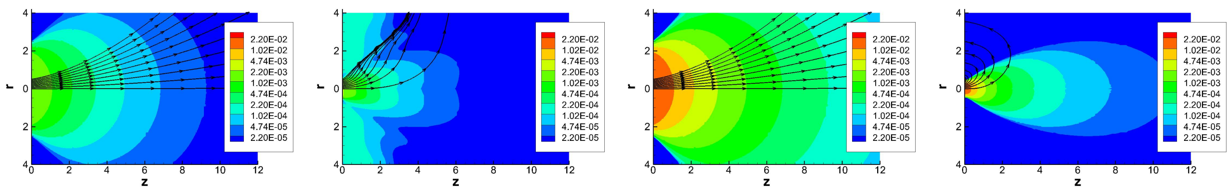

Figure 5. Axial magnetic field contours with magnetic field streamlines. First: Initial magnetic field for $B_0 = 0.002$ (T), **Second: Final magnetic field for** $B_0 = 0.002$ (T), **Third: Initial magnetic field for** $B_0 = 0.02$ (T), **Fourth: Final magnetic field for** $B_0 = 0.02$ (T)

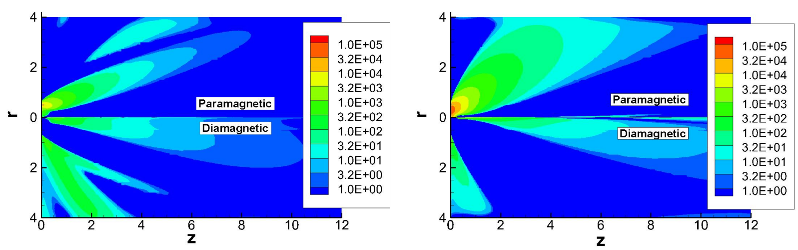

Figure 6. Induced currents, J (A/m^2), **in** $\hat{\theta}$ **direction. Top half contours are paramagnetic (positive** $\hat{\theta}$**) and bottom half contours are diamagnetic (negative** $\hat{\theta}$**).**

C. Generalized Ohm's Law Results

The solver used for the generalized Ohm's law study is the Magneto - Gas Kinetic Method (MGKM).[38,47,50] This is a 3D Cartesian MHD solver which incorporates a generalized Ohm's law. In these simulations the plasma enters the domain through an inlet and flows into a plasma at the same static pressure. There is no jet expansion in these cases. The boundary around the inlet is an inviscid wall while the far-field boundaries have Neumann boundary conditions applied. The applied magnetic field is generated by a current loop outside of the domain. The goal of these simulations is to study the effect of the Hall term on the behavior of MHD jets under the influence of a magnetic nozzle. The fluid in this simulation is similar to a Xenon plasma used in electric propulsion devices. The flow parameters are given in Table 3. The domain is a 200x64x64 grid with $\Delta x = 0.025$.

Gerwin[15,16] predicts that the primary effect of the Hall term on a magnetic nozzle plasma jet would be to induce an azimuthal, rotational velocity. The Hall term describes the effects of the demagnetization of ions. Ions may then travel across magnetic field lines creating axial and radial currents which result in azimuthal forces that rotate the plasma jet. Figure 7 shows axial currents that develop due to Hall term effects. These currents result in the $\boldsymbol{J} \times \boldsymbol{B}$ force which rotates the jet as evidenced by the azimuthal velocity and streamline

Case	ρ (kg/m^3)	n $(\#/m^3)$	T (eV)	U_{in} (m/s)	μ $(kg \cdot m/s)$	σ (S/m)	B (T)
Num. Exp.	$1.0 \cdot 10^{-5}$	$4.59 \cdot 10^{19}$	$3.04 \cdot 10^{0}$	$4.1 \cdot 10^{2}$	$1.37 \cdot 10^{-6}$	$1.0 \cdot 10^{5}$	$2.0 \cdot 10^{-3}$

Case	Reynolds	Magnetic Reynolds	Mach	Electron Hall	$1/(\tau_{res}\omega_{ci})$	p_{back}/p_{jet}
Num. Exp.	$6.00 \cdot 10^{2}$	$1.03 \cdot 10^{0}$	$2.17 \cdot 10^{-1}$	$2.72 \cdot 10^{0}$	$1.74 \cdot 10^{-1}$	1

Table 3. Fluid conditions for generalized Ohm's law simulation

rotation shown in Figure 8. Interaction of the magnetic field with the radial induced currents also contributes to this rotation. The azimuthal velocity primarily appears in the region of axial current, giving confidence that this is the cause for jet rotation. Therefore, comb the Hall effect generates helical structures in both the velocity and current by creating azimuthal velocities and longitudinal ($\hat{r}$ and $\hat{z}$) currents. More thorough discussion of these results can be found in a previous paper.[38]

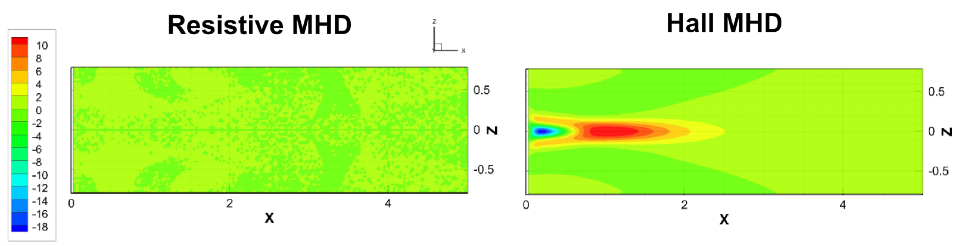

Figure 7. Contours of axial ($\hat{x}$) current

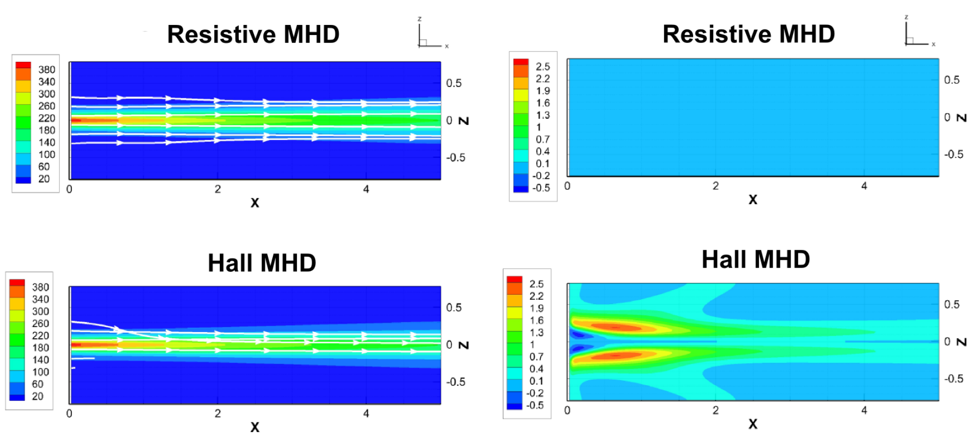

Figure 8. Left: Contours of axial velocity and velocity streamlines, Right: Contours of azimuthal ($\hat{\theta}$) velocity (m/s)

IV. Conclusion

Preliminary results with resistive and generalized Ohm's law MHD solvers are presented. Simple resistive MHD simulations are performed which generate results which, as expected, do not agree well with experimental results in the desired regimes and further development is required. Numerical challenges for future

MHD simulations are identified in this study. The addition of the Hall term generates an azimuthal rotation of the plasma jet in a magnetic nozzle as predicted in theory. More generally, helical structures in both the current and velocity field are generated in the magnetic nozzle plume due to the Hall effect.

Future work will continue development of generalized Ohm's law MHD solvers and will also investigate the use of other computational methods such as hybrid PIC methods for study of magnetic nozzle physics.

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