Development of a 2D Axial-Radial Fluid Electron Model

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Nested Channel Hall Effect Thrusters

- Hall Effect Thrusters (HETs) have a rich history of over 60 years [1]
- Nested channel HETs were first developed at the University of Michigan in the Plasmadynamics and Electric Propulsion Laboratory (PEPL):
  - 2 channel, 10kW class X2 by Liang [1]
  - 3 channel, 100kW class X3 by Florenz [2]

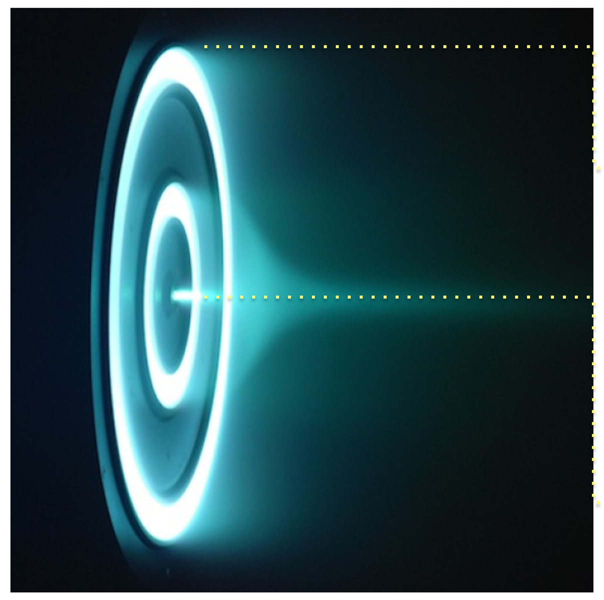
Figure 1: X2 NHT [1].

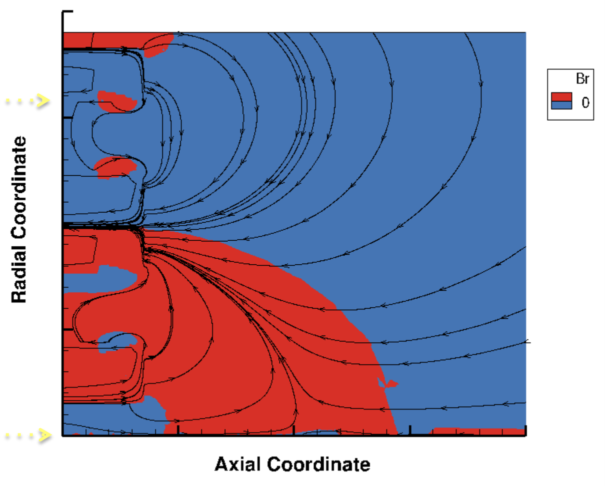

Figure 2: Radial magnetic field map.

Performance gains were observed during multiple channel operation [1]

Quasi 1D Electron Model

- Thermalized potential approximation
- Based on difference in mobility along and across B field lines
- Equations written in terms of only one variable, the magnetic stream function
- only works for simple B fields and geometries

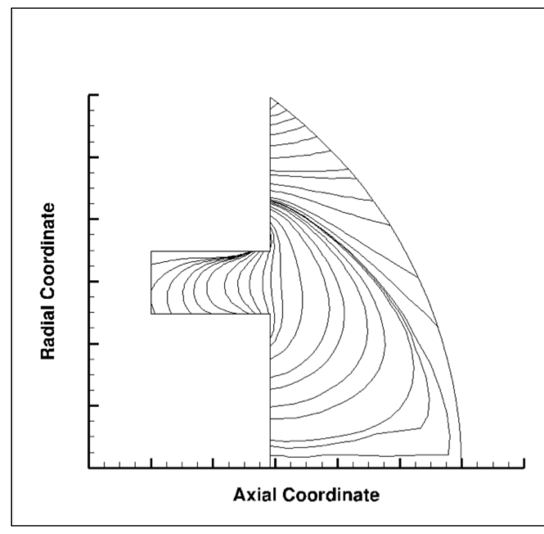

Figure 3: Quasi – 1D domain [3], [4].

Fully 2D Electron Model

Generalized Ohm's law:

$$\vec{E} = \frac{q\vec{\Gamma}_e}{\sigma} - \frac{q\vec{\Gamma}_e \times \vec{B}}{qn_e} + \frac{\nabla \cdot \underline{\underline{p}}}{qn_e}$$

Electron momentum, in terms of flux:

$$\frac{\partial \Gamma_{er}}{\partial r} + \frac{\partial \Gamma_{ez}}{\partial z} + \frac{\Gamma_{er}}{r} = 0$$

Expansion in terms of potential and pressure:

$$LHS = \frac{\partial}{\partial z}\left(Z_1\frac{\partial \phi}{\partial z} + Z_2\frac{\partial \phi}{\partial r}\right) + \frac{\partial}{\partial r}\left(R_1\frac{\partial \phi}{\partial z} + R_2\frac{\partial \phi}{\partial r}\right) + \left(\frac{R_1\frac{\partial \phi}{\partial z} + R_2\frac{\partial \phi}{\partial r}}{r}\right)$$

$$RHS = -\left(\frac{\partial Z_3}{\partial z} + \frac{\partial R_3}{\partial r} + \frac{R_3}{r}\right)$$

$$= -\left[\frac{\partial}{\partial z}\left(ZP_1\frac{\partial p_e}{\partial z} + ZP_2\frac{\partial p_e}{\partial r}\right) + \frac{\partial}{\partial r}\left(RP_1\frac{\partial p_e}{\partial z} + RP_2\frac{\partial p_e}{\partial r}\right) + \frac{RP_1\frac{\partial p_e}{\partial z} + RP_2\frac{\partial p_e}{\partial r}}{r}\right]$$

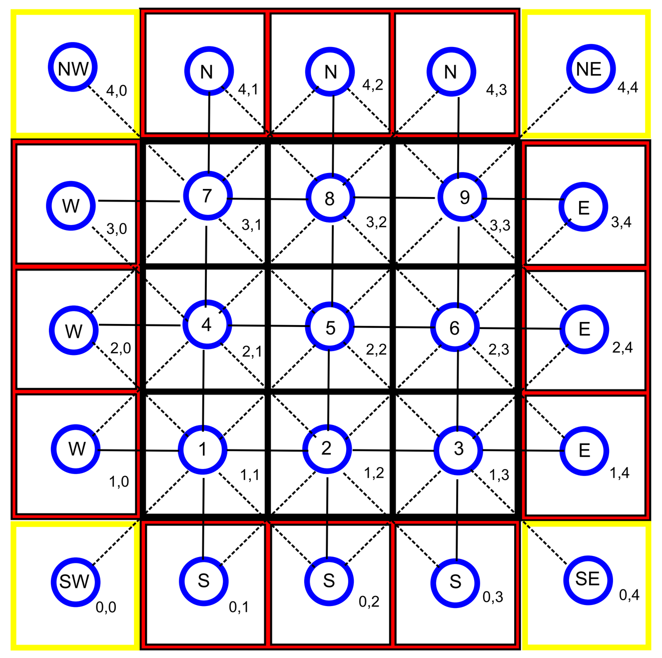
Figure 4: 2D domain.

Inputs

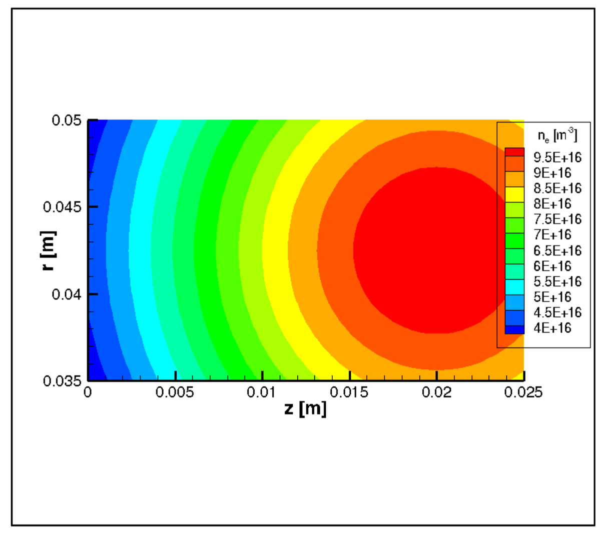
Figure 5: Electron number density.

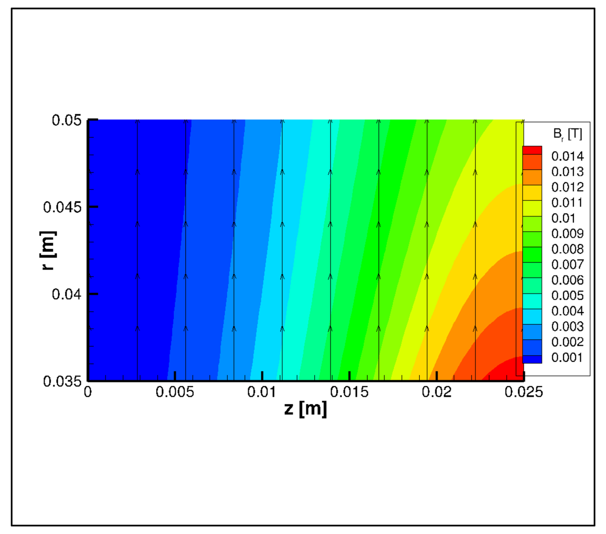
Figure 7: Br, α=0

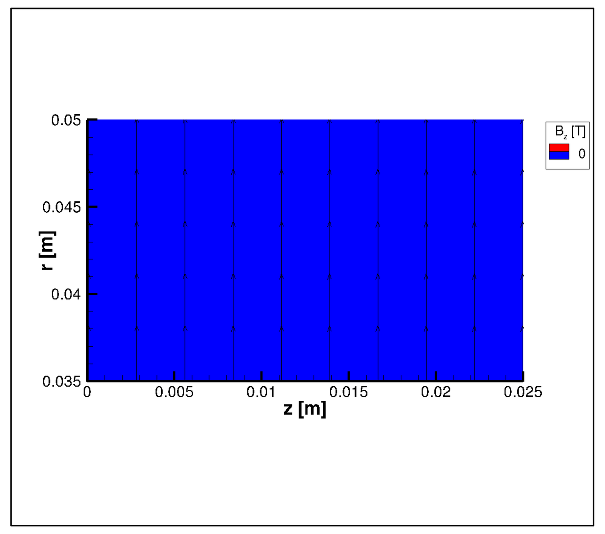
Figure 8: Bz, α=0

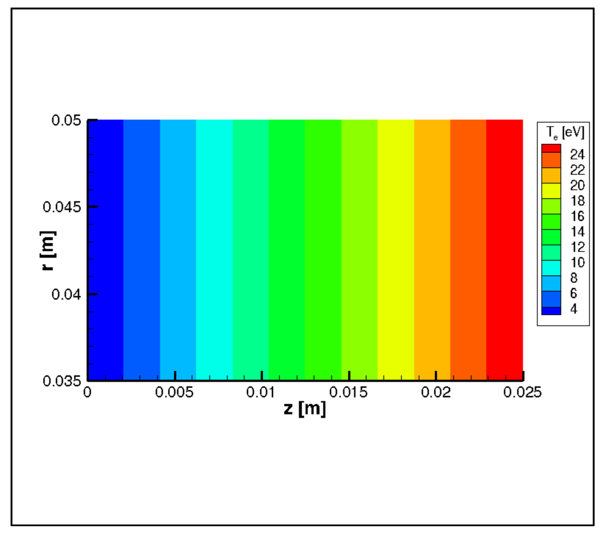
Figure 6: Electron temperature.

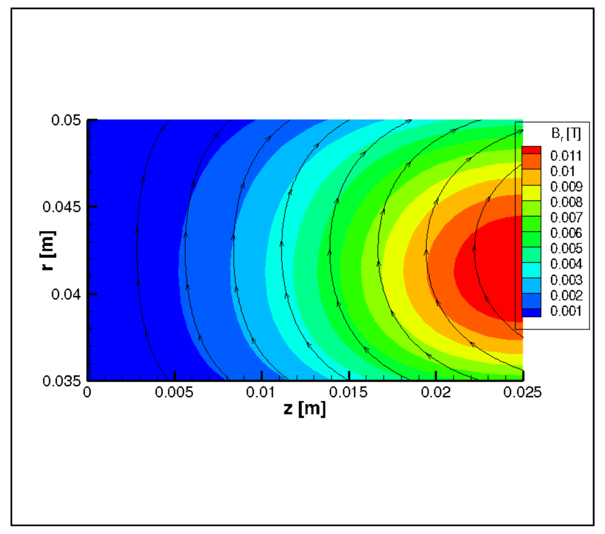
Figure 9: Br, α=-10,000

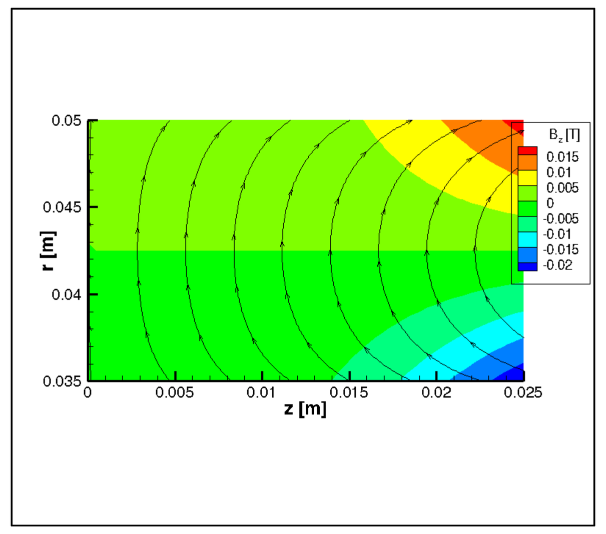
Figure 10: Br, α=-10,000

Plasma Potential

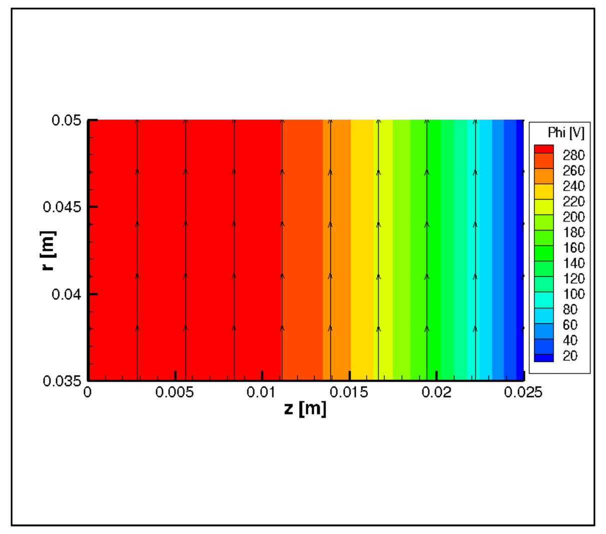
Figure 11: α=0, p=0

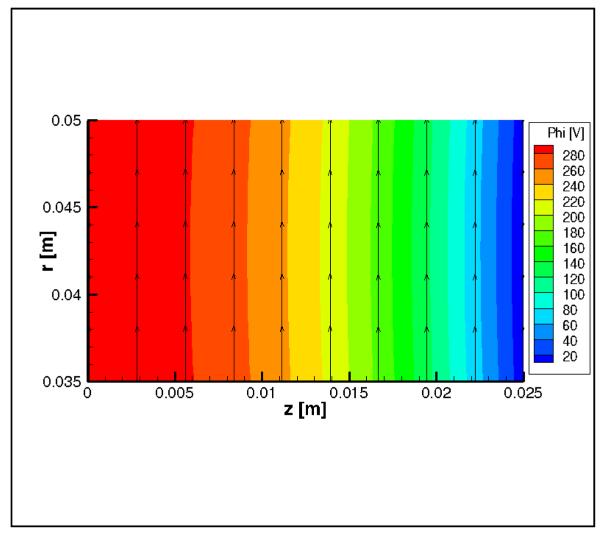
Figure 12: α=0, p nonuniform

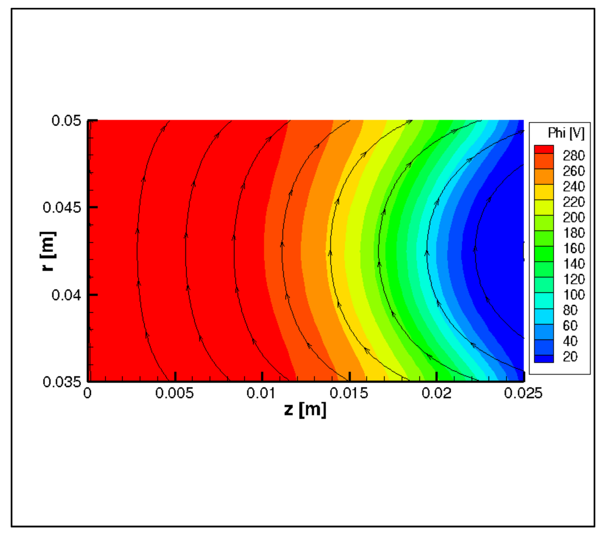
Figure 13: α=-10,000, p=0

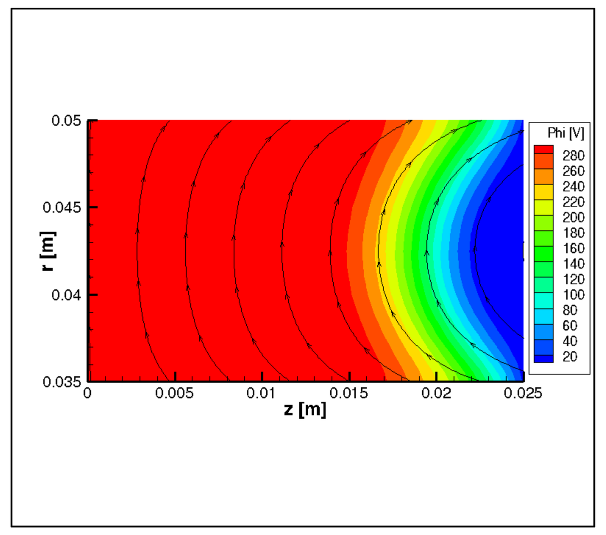
Figure 14: α=-10,000, p nonuniform

Future Work

Potential Solver:

- Verification: with analytic solution
- Validation: compare to experimental measurements (H6, X2)
- Benchmarking: consistent comparison with HPHall

Electron Energy:

- Initially use Te from quasi-1D formulation
- Consider using a 2 equation approach

Coupling with hybrid PIC code HPHall:

- PIC for heavy species
- Replace quasi 1D potential solver with 2D model for electrons
- Simulate X2 dual channel, including plume

References

1.Liang, R., "The Combination of Two Concentric Discharge Channels into a Nested Hall-Effect Thruster," Ph.D. Dissertation, Aerospace Engineering Dept., University of Michigan., Ann Arbor, MI, 2013.
2.Florenz, R.E., "The X3 100-kW Class Nested-Channel Hall Thruster: Motivation, Implementation and Initial Performance," Ph.D. Dissertation, Aerospace Engineering Dept., University of Michigan., Ann Arbor, MI, 2014.
3.Fife, J.M., "Hybrid-PIC Modeling and Electrostatic Probe Survey of Hall Thrusters," Ph.D. Dissertation, Dept. of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, 1998.
4.Hofer, R. R., et al., "Efficacy of Electron Mobility Models in Hybrid-PIC Hall Thruster Simulations," AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, AIAA 2008-4924, Hartford, CT, July 21-23, 2008.

Acknowledgments

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